Characterization of Aloha in Underwater Wireless Networks

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Abstract—This paper provides an analytic framework of Aloha and slotted Aloha (S-Aloha) performance in an underwater one-to-one communication environment with high and random inter-nodal signal propagation delay. The analysis shows that random inter-nodal propagation delay has no effect on the underwater Aloha performance. It also sheds light on the throughput degradation of underwater S-Aloha with a slotting concept that achieves terrestrial S-Aloha equivalent one-slot vulnerability. Additionally, a new, modified slotting concept is introduced where the slot size is judiciously reduced such that even by allowing some collisions the overall system throughput can be increased. Our numerical and simulation results show that, with the modified slotting approach up to 17% throughput performance gain can be achieved over the naive (terrestrial S-Aloha equivalent) slotting approach.

I. INTRODUCTION

Underwater wireless networks (UWN) and terrestrial wireless networks differ in many aspects; propagation delay is the most sensitive parameter of them all. Terrestrial radio frequency (RF) networks universally use electromagnetic (EM) waves. However, due to high attenuation, underwater (UW) wireless networks cannot use EM waves; instead acoustic waves are used. The bandwidth of UWN is also a few orders of magnitude lower than that of the RF wireless networks. This clearly means that the protocols designed for RF networks are unlikely to be directly applicable in UWN [1]–[3].

There have been some recent works on UWN multiaccess networks. Based on simulation studies of UWN it was suggested in [4] that the maximum performance of S-Aloha is the same as that of Aloha, and the S-Aloha performance further degrades with increase in propagation delay. The effects of inter-nodal propagation delay on many-to-one Aloha and S-Aloha throughput performance was studied via simulations in [5]. The Aloha performance was shown to be unaffected by spatial uncertainty. With a slot size equal to a (fixed) frame transmission time, their simulation results on S-Aloha showed throughput degradation with increase in propagation delay. Further, to enhance the S-Aloha performance, the authors proposed to increase the slot size by some fractional amount. An analytic study of the many-to-one protocols proposed in [5] was performed in [6]. In [7], two Aloha based variants were proposed, where, a node upon overhearing the neighboring nodes' communication, takes appropriate backoff measure so as to minimize the collision probability.

To counter the effect of UW propagation delay, RTS/CTS (request-to-send/clear-to-send) based reservation protocol was proposed in [8], where based on the propagation delay of RTS frame and the data length information in it, the receiving

node decides a receive window for a collision-free data frame reception. In another work [9], communication between a master (gateway) node and the slave (non-gateway) nodes was considered, where separate channels for control (reservation) and data were suggested in RTS/CTS handshake based reservation protocol. The RTS frames from the non-gateway nodes are sent using the Aloha protocol, and until a desired CTS frame is received at a non-gateway node, it does not transmit its data frame. Note that, such schemes are efficient with relatively longer frames and infrequent transmissions. This process also ensures collision-free data transmission in a single-cell scenario. However, when smaller frames comparable to the size of RTS-CTS frames are transmitted frequently, such explicit reservation mechanisms are clearly not efficient.

Thus, while reservation based multiaccess protocols, such as CSMA/CA with RTS/CTS, may offer a higher throughput, basic Aloha protocols would be of interest in situations where the return channel for reservation is unavailable or infeasible to use. In other words, basic Aloha protocols are expected to be used in UW communications for short frame transmissions or as a reservation protocol for supporting longer sessions (as in [9], similar to the contention-based channel access in wireless LANs and for paging in the GSM cellular systems).

This work focuses on theoretically characterizing the throughput performance of the basic Aloha protocols for oneto-one communications in UWN. Our contributions in this paper are as follows: (a) We derive generalized throughput performance expression for Aloha and show that its performance is indeed independent of signal propagation speed. (b) Noting that the throughput of terrestrial S-Aloha equivalent S-Aloha-uw (i.e., with one slot vulnerability) is dictated by the maximum propagation delay within the nodal coverage range, we propose an aggressive slotting where for a given nodal communication range, slot size can be appropriately chosen as a function of the frame size. Via a closed form analysis supported by simulations we demonstrate that, an optimal choice of slot size can lead up to 17% throughput performance gain with respect to the naive slotting decision.

The objectives in this paper match closely with that of [4], [5], and [6]. However, in contrast with these studies, we provide analyses of *one-to-one* Aloha-uw and S-Aloha-uw for *any* value of inter-nodal propagation delay. Our analysis approach is different from that provided in [6] for many-to-one Aloha protocols. The slotting approach in S-Aloha-uw proposed in this paper is different in that, instead of one frame transmission time T_t as the slot size, we propose to have a slot size which is the sum of T_t and the maximum inter-nodal

propagation delay T_p^{\max} , where T_p^{\max} can be of any value such that $T_p^{\max} < \text{or}$, = or, $> T_t$. To increase the throughput efficiency of S-Aloha-uw, we propose and analyze an optimum slot size reduction factor k.

General assumptions and major notations are provided in Section II. Aloha-uw throughput is analyzed in Section III. Section IV describes our S-Aloha-uw slotting concept and the throughput analysis. Our proposed modified S-Aloha-uw is presented and analyzed in Section V. Performance results are discussed in Section VI. Section VII concludes the paper.

II. GENERAL ASSUMPTIONS AND NOTATIONS

- 1) The network consists of homogeneous nodes.
- 2) Nodes are uniformly randomly distributed.
- 3) The traffic arrival process is Poisson distributed.
- 4) Physical channel related frame errors are not considered.
- 5) Temporal variability of inter-nodal propagation delay due to UW current is not accounted.
- 6) Network performance is measured in terms of *normalized system throughput*, defined as the average number of successful frames per frame transmission time.

Major notations used in the paper are listed in Table I:

TABLE I

SUMMARY OF NOTATIONS	
R	Nodal communication (transmit/receive) range
v	Acoustic signal propagation speed
c	RF signal propagation speed
λ	Frame arrival rate in the system per unit time
η	Normalized system (network) throughput
F	Frame length
R_c	Channel rate
T_t	Frame transmission time; $T_t = \frac{F}{R_c}$
T_p	Signal propagation delay, a function of transmitter-receiver
	distance r ; $T_p = \frac{r}{v}$; $r \leq R$
T_{n}^{\max}	Maximum signal propagation delay; $T_n^{\max} = \frac{R}{n}$
F	
$T_s^{\mathbf{rf}}$	Slot size in S-Aloha-rf; $T_s^{\rm rf} = T_t$
$\frac{T_s^{\rm rf}}{T_s^{\rm uw}, T_{s1}}$	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified
$\frac{T_s^{\rm ff}}{T_s^{\rm uw}, T_{s1}}$	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{uw}} = T_t + T_p^{\text{max}} = T_{s1}$
$\frac{T_s^{\text{ff}}}{T_s^{\text{uw}}, T_{s1}}$ $\frac{T_{sk}}{T_{sk}}$	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{UW}} = T_t + T_p^{\text{max}} = T_{s1}$ Slot size in mS-Aloha-uw (modified S-Aloha-uw)
$\frac{T_s^{\text{rf}}}{T_s^{\text{uw}}, T_{s1}}$ T_{sk}	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{UW}} = T_t + T_p^{\text{max}} = T_{s1}$ Slot size in mS-Aloha-uw (modified S-Aloha-uw) with $0 \le k \le 1$; $T_{sk} = T_t + kT_p^{\text{max}}$
$\frac{T_s^{\rm rf}}{T_s^{\rm uw}, T_{s1}}$ $\frac{T_{sk}}{k}$	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{UW}} = T_t + T_p^{\text{max}} = T_{s1}$ Slot size in mS-Aloha-uw (modified S-Aloha-uw) with $0 \le k \le 1$; $T_{sk} = T_t + kT_p^{\text{max}}$ Slot size reduction factor in mS-Aloha-uw; $0 \le k \le 1$
$\frac{T_s^{\text{rf}}}{T_s^{\text{uw}}, T_{s1}}$ $\frac{T_{sk}}{T_p}$	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{UW}} = T_t + T_p^{\text{max}} = T_{s1}$ Slot size in mS-Aloha-uw (modified S-Aloha-uw) with $0 \le k \le 1$; $T_{sk} = T_t + kT_p^{\text{max}}$ Slot size reduction factor in mS-Aloha-uw; $0 \le k \le 1$ Distance of the receiver from a neighboring transmitter,
$\frac{T_{s}^{\text{rf}}}{T_{s}^{\text{uw}}, T_{s1}}$ $\frac{T_{sk}}{T_{p}}$	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{UW}} = T_t + T_p^{\text{max}} = T_{s1}$ Slot size in mS-Aloha-uw (modified S-Aloha-uw) with $0 \le k \le 1$; $T_{sk} = T_t + kT_p^{\text{max}}$ Slot size reduction factor in mS-Aloha-uw; $0 \le k \le 1$ Distance of the receiver from a neighboring transmitter, that has a frame in <i>previous</i> slot
$\frac{T_{s}^{\text{rf}}}{T_{s}^{\text{uw}}, T_{s1}}$ $\frac{T_{sk}}{T_{p}}$ r_{n}	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{UW}} = T_t + T_p^{\text{max}} = T_{s1}$ Slot size in mS-Aloha-uw (modified S-Aloha-uw) with $0 \le k \le 1$; $T_{sk} = T_t + kT_p^{\text{max}}$ Slot size reduction factor in mS-Aloha-uw; $0 \le k \le 1$ Distance of the receiver from a neighboring transmitter, that has a frame in <i>previous</i> slot Distance of the receiver from a neighboring transmitter,
$\frac{T_s^{\text{rf}}}{T_s^{\text{uw}}, T_{s1}}$ $\frac{T_{sk}}{T_p}$ r_n	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{UW}} = T_t + T_p^{\text{max}} = T_{s1}$ Slot size in mS-Aloha-uw (modified S-Aloha-uw) with $0 \le k \le 1$; $T_{sk} = T_t + kT_p^{\text{max}}$ Slot size reduction factor in mS-Aloha-uw; $0 \le k \le 1$ Distance of the receiver from a neighboring transmitter, that has a frame in <i>previous</i> slot Distance of the receiver from a neighboring transmitter, that has a frame in <i>next</i> slot
$ \begin{array}{c} T_s^{\mathrm{rf}} \\ T_s^{\mathrm{rw}}, T_{s1} \\ T_{sk} \\ k \\ r_p \\ r_n \\ n_p \end{array} $	Slot size in S-Aloha-rf; $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-rg, $T_s^{\text{rf}} = T_t$ Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\text{uW}} = T_t + T_p^{\text{max}} = T_{s1}$ Slot size in mS-Aloha-uw (modified S-Aloha-uw) with $0 \le k \le 1$; $T_{sk} = T_t + kT_p^{\text{max}}$ Slot size reduction factor in mS-Aloha-uw; $0 \le k \le 1$ Distance of the receiver from a neighboring transmitter, that has a frame in <i>previous</i> slot Distance of the receiver from a neighboring transmitter, that has a frame in <i>next</i> slot Number of frames scheduled in <i>previous</i> slot

III. ALOHA IN UWN

In this section, we analyze the pure Aloha protocol performance in UWN considering fixed frame size. The case of variable frame size is omitted due to space constraints.

In UWN, there is an appreciable signal propagation delay T_p compared to T_t . Since collision happens at the receiver, a frame of size T_t , whose reception starts at time $t + T_p$, will be successful if no additional arrival occurs during the interval $2T_t$ (from $t + T_p - T_t$ to $t + T_p + T_t$), even though the possible arrivals during this time could be caused by the generation process over a larger time duration (which is $2T_t + T_p^{\text{max}}$ in case of UWN). This concept is depicted in Fig. 1.



Fig. 1. Pictorial representation of collision vulnerability concept.

Consider the number of frames that arrive in window $[t_2, t_3] = m$, and the generated ones during $[t_1, t_3] = n$. Irrespective of the signal propagation delay, i.e., for any internodal communication range, we have [10, Ch. 3]

$$\Pr\left[m \text{ out of } n \text{ frames arrive during } [t_2, t_3]\right]$$
$$\stackrel{\Delta}{=} P_n(m) = \binom{n}{m} p^m (1-p)^{n-m}, \qquad (1)$$

where $p = \frac{|t_2 - t_3|}{|t_1 - t_3|} = \frac{2T_t}{|t_1 - t_3|}$. Since the frame generation process in the system is Poisson, the arrivals in the window $[t_2, t_3]$ can also be approximated as Poisson distributed, as follows. The frame arrival rate in the system is $\lambda = \frac{n}{|t_1 - t_3|}$. In case of a homogeneous frame generation process, the window $|t_1 - t_3|$ can be increased arbitrarily, leading to $n \to \infty$ and $p \to 0$, keeping the product $np = 2\lambda T_t$ a constant. Accordingly, (1) can be approximated as:

$$P_n(m) \approx e^{-np} \frac{(np)^m}{m!} = e^{-2\lambda T_t} \frac{(2\lambda T_t)^m}{m!}.$$
 (2)

The frame success probability is, $P_n(0) = e^{-2\lambda T_t}$. Hence, the normalized system throughput with fixed frame size is:

$$\eta_{\text{Aloha-uw}}^{\text{(fixed)}} = \lambda T_t e^{-2\lambda T_t},\tag{3}$$

which is the same as the Aloha-rf throughput.

IV. SLOTTED ALOHA IN UWN

We have the throughput expression for S-Aloha-rf as:

$$\eta_{\text{S-Aloha-rf}} = \lambda T_t e^{-\lambda T_t}.$$
(4)

In a UWN, propagation delay T_p of a frame to the receiver varies between 0 and T_p^{max} (see Fig. 2(b) and (c)). Since the synchronization in a slotted access protocol is done at the transmitter nodes, to resemble the one-slot S-Aloha vulnerability concept as in short-range RF communications, a buffer time T_p^{max} is needed. Thus, unlike in S-Aloha-rf, where the slot size is $T_s^{rf} = T_t$ (see Fig. 2(a)), the slot size in S-Alohauw should be $T_s^{uw} = T_t + T_p^{max} \triangleq T_{s1}$ (see Fig. 2(c)). Also,



Fig. 2. Slotting concepts. (a) Slot size in S-Aloha-rf; (b) effect of signal propagation delay on the time lag between a frame transmission and its reception process; (c) slot size in S-Aloha-uw.

S-Aloha-rf like frame success probability is achieved as long as $T_p^{\max} < T_t$. However, if $T_p^{\max} \ge T_t$, more than one frame generated in a slot do not necessarily cause a frame collision at

the receiver. So, the throughput for one-to-one communication has to be computed differently for the two regimes. Case 1: $T_p^{\max} < T_t$

The throughput computation in this regime is done similarly as in S-Aloha-rf. The normalized system throughput is:

$$\eta_{\text{S-Aloha-uw}}(T_p^{\max} < T_t) = \lambda T_t e^{-\lambda \left(T_t + T_p^{\max}\right)}.$$
 (5)

Case 2: $T_p^{\max} \ge T_t$

Let the receiver's distance from its intended transmitter be $\mathbf{r} = r$. The frame success probability P_S is obtained as:

$$P_S = \int_{r=0}^{R} \Pr[\operatorname{success}|\mathbf{r}=r] \cdot \Pr[\mathbf{r}=r].$$
(6)

If a transmitter-receiver distance is uniformly random, the density function (pdf) of \mathbf{r} is:

$$f_{\mathbf{r}}(r) = \begin{cases} \frac{2r}{R^2}, & 0 \le r \le R\\ 0, & \text{elsewhere.} \end{cases}$$
(7)

Hence,

$$\Pr[\mathbf{r} = r] \equiv \Pr[r \le \mathbf{r} \le r + dr] = f_{\mathbf{r}}(r)dr = \frac{2rdr}{R^2}.$$
 (8)

The regime of $T_p^{\max} \ge T_t$ is further subdivided into two. Case 2-a: $T_t \le T_p^{\max} \le 2T_t$

There are three sub-regions of \mathbf{r} . In *sub-region* 1, where $0 \leq \mathbf{r} \leq R - T_t v$, if n additional frames are generated in the same slot, the intended frame will be successful as long as all n other frames have propagation delay $\mathbf{T'_p} \geq \frac{r}{v} + T_t$. Since $\mathbf{T'_p} = \frac{\mathbf{r'}}{v}$, where $\mathbf{r'}$ is a random variable (RV) representing the distance of a neighboring transmitter from the intended receiver, the above condition reduces to $\mathbf{r'} \geq r + T_t v$. Hence, the conditional frame success probability is obtained as:

$$P_{S_{1a}} = e^{-\lambda T_{s1} \left(\frac{r+T_t v}{R}\right)^2},\tag{9}$$

where, from (7),

$$\Pr[\mathbf{r}' \le r + T_t v] = \begin{cases} \frac{(r + T_t v)^2}{R^2}, & 0 \le r \le R - T_t v\\ 1, & R - T_t v \le r \le R \\ 0, & \text{elsewhere.} \end{cases}$$
(10)

In sub-region 2, where $R - T_t v < \mathbf{r} < T_t v$, the frame will be successful if there are no additional frames generated from any neighboring transmitters in the same slot. Accordingly,

$$P_{S_{2a}} = e^{-\lambda T_{s1}}.$$
 (11)

In sub-region 3, where $T_t v \leq \mathbf{r} \leq R$, the intended frame will be successful if there are n additional frames in the same slot generated at distance \mathbf{r}'' such that $0 \leq \mathbf{r}'' \leq r - T_t v$. The conditional frame success probability is obtained as:

$$P_{S_{3a}} = e^{-\lambda T_{s1} \left[1 - \left(\frac{r - T_t v}{R} \right)^2 \right]},$$
 (12)

where

$$\Pr[0 \le \mathbf{r}'' \le r - T_t v] = \begin{cases} \frac{(r - T_t v)^2}{R^2}, & T_t v \le r \le R\\ 0, & 0 \le r \le T_t v \\ 1, & \text{elsewhere.} \end{cases}$$
(13)

Using (6), the frame success probability P_{S_a} is obtained as:

$$P_{Sa} = \int_{r=0}^{R-T_t v} P_{S_{1a}} \frac{2rdr}{R^2} + \int_{R-T_t v}^{T_t v} P_{S_{2a}} \frac{2rdr}{R^2} + \int_{T_t v}^{R} P_{S_{3a}} \frac{2rdr}{R^2}.$$
(14)

Hence, the normalized system throughput is:

$$\eta_{\text{S-Aloha-uw}}(T_t \le T_p^{\max} \le 2T_t) = \lambda T_t P_{S_a}.$$
(15)

Case 2-b: $T_p^{\max} > 2T_t$

In sub-region 1, $0 \leq \mathbf{r} \leq T_t v$, $P_{S_{1b}}$ is given by (9).

In sub-region 2, $T_t v < \mathbf{r} < R - T_t v$. If *n* additional frames from neighboring transmitters are generated, of which n' are from a distance \mathbf{r}' such that $R \ge \mathbf{r}' \ge r + T_t v$ and n - n' are from a distance \mathbf{r}'' such that $0 \le \mathbf{r}'' \le r - T_t v$, the intended frame to the receiver will still be successful. Thus,

$$P_{S_{2b}} = \sum_{n=0}^{\infty} \sum_{n'=0}^{n} \left[1 - \left(\frac{r+T_t v}{R}\right)^2 \right]^{n'} \left(\frac{r-T_t v}{R}\right)^{2(n-n')} \\ \cdot \frac{\left(\lambda T_{s1}\right)^n}{n!} e^{-\lambda T_{s1}}.$$
 (16)

In sub-region 3, $R - T_t v \leq \mathbf{r} \leq R$, $P_{S_{3b}}$ is given by (12). Combining, the frame success probability is given by

$$P_{S_b} = \int_{r=0}^{T_t v} P_{S_{1b}} \frac{2rdr}{R^2} + \int_{T_t v}^{R-T_t v} P_{S_{2b}} \frac{2rdr}{R^2} + \int_{R-T_t v}^{R} P_{S_{3b}} \frac{2rdr}{R^2}.$$
(17)

Hence, the normalized system throughput is obtained as:

$$\eta_{\text{S-Aloha-uw}}(T_p^{\max} > 2T_t) = \lambda T_t P_{S_b}.$$
 (18)

V. A NEW SLOTTING CONCEPT FOR S-ALOHA IN UWN

From the analysis in Section IV it can be noted that, with the naive slotting concept in S-Aloha-uw, the slot size has to be larger than that of S-Aloha-rf by $T_p^{\max} = \frac{R}{v}$. In most cases, however, a transmitter-receiver distance r is less than R, and so the reception is completed before the slot ends (see Fig. 2(b)). Note that, in one-to-one communication, after the frame reception at a node is completed, the system remains idle for the duration $T_p^{\max} - T_p$, thereby reducing in system throughput. It is also clear from (5) that, for a given λ , the higher the ratio $\frac{T_p^{\max}}{T_t}$, the lesser the system throughput $\eta_{\text{S-Aloha-uw}}$ compared to $\eta_{\text{S-Aloha-rf}}$ in (4). Similar trends are expected at $T_p^{\max} \geq T_t$ (see (15) and (18)), which are presented in Section VI. For one-to-one communication, other than having reduced system throughput, no additional intuition is derived from the cases of $T_p^{\max} \geq T_t$. So, we will restrict our further studies on S-Aloha-uw for $T_p^{\max} < T_t$.

Since it is likely that in many cases r < R, it may be wise to reduce the slot size optimally so as to minimize the system idling time without increasing the collision vulnerability, thereby increasing the system throughput. We call this modified slotted Aloha protocol as mS-Aloha-uw. The collision behavior of mS-Aloha-uw $T_p^{\max} < T_t$ is shown in Fig. 3. In this slotting approach, the extra time to accommodate the random propagation delay is reduced to kT_p^{\max} , where k $(0 \le k \le 1)$ is the *slot size reduction factor*. The modified total slot size $T_{sk} = T_t + kT_p^{\max} \le T_{s1}$. Note that, k = 0



Fig. 3. Modified slotting concept in UWN. $T_{sk} = T_t + kT_p^{\text{max}}$, where $0 \le k \le 1$ and $T_p^{\max} < T_t$. (a) A frame from a r_1 distance away transmitter scheduled in the previous slot may cause collision with a frame in the current slot if $kR < r_1 < R$. (b) A frame in the current slot from a r_2 distance away transmitter may encounter collision with a frame scheduled in the next slot if $kR < r_2 < R$. (c) A frame scheduled from a r_3 distance away transmitter, $0 < r_3 < kR$, does not cause collision with the frames in other slots.

corresponds to the slot size in S-Aloha-rf, whereas k = 1corresponds to the naive S-Aloha-uw.

mS-Aloha-uw throughput is computed using the general expression in (6), where the RV r is the intended receiver's distance to the transmitter that has a frame scheduled in the current slot (slot i). However, in addition to a collision probability due to more than one frame scheduled in slot *i*, two more conditions exist. For k < 0.5, a frame transmitted in slot *i* can be vulnerable simultaneously due to the neighboring nodes' transmissions in the two adjacent slots i-1 and i+1; whereas, for $k \ge 0.5$, vulnerability of a frame can be caused by a transmission in either the previous slot i - 1 or the next slot i + 1. Accordingly, the probability of successful reception of a frame in mS-Aloha-uw is computed differently for $0 \le k \le 0.5$ and for $0.5 \le k \le 1.0$.

Case 1: $0 \le k \le 0.5$

In this range, the frame success probability is given by:

$$P_S(0 \le k \le 0.5) \stackrel{\Delta}{=} P'_{S_1} + P'_{S_2} + P'_{S_3} \tag{19}$$

Note that, in addition to counting the possibility of more than one scheduled frames in slot i, P'_{S_1} captures the vulnerability due to the scheduled frames in slot i - 1, P'_{S_2} absorbs the vulnerability due to the scheduled frames in slot i - 1 as well as slot i+1, whereas P'_{S_3} accommodates the vulnerability due to the scheduled frames in slot i + 1.

Using (8) and (10), P'_{S_1} in (19) is obtained as:

$$P_{S_{1}}^{\prime} = \frac{e^{-2\lambda T_{sk}}}{\lambda T_{sk}} \left[e^{4\lambda T_{sk}k^{2}} - e^{\lambda T_{sk}k^{2}} \right] - \frac{2ke^{-2\lambda T_{sk}}}{\sqrt{\lambda T_{sk}}}$$
(20)

$$\cdot \left[e^{4\lambda T_{sk}k^{2}} D_{+} \left(2k\sqrt{\lambda T_{sk}} \right) - e^{\lambda T_{sk}k^{2}} D_{+} \left(k\sqrt{\lambda T_{sk}} \right) \right],$$

where $D_+(x) = e^{-x^2} \int_0^x e^{t^2} dt$ is the Dawson's integral [11].

Using (8), (10), and (13), P'_{S_2} is obtained as:

$$P'_{S_2} = \frac{2e^{-2\lambda T_{sk}}}{(4\lambda T_{sk}k)^2} \left[\left(4\lambda T_{sk}k(1-k) - 1 \right) e^{4\lambda T_{sk}k(1-k)} - \left(4\lambda T_{sk}k^2 - 1 \right) e^{4\lambda T_{sk}k^2} \right].$$
(21)

Similarly, the expression for P'_{S_3} is given by:

$$P'_{S_3} = \frac{e^{-\lambda T_{sk}}}{\lambda T_{sk}} \left[e^{-\lambda T_{sk}(1-2k)^2} - e^{-\lambda T_{sk}(1-k)^2} \right] + \frac{\sqrt{\pi k} e^{-\lambda T_{sk}}}{\sqrt{\lambda T_{sk}}} \\ \cdot \left[\operatorname{erf} \left(\sqrt{\lambda T_{sk}} (1-k) \right) - \operatorname{erf} \left(\sqrt{\lambda T_{sk}} (1-2k) \right) \right],$$
(22)

where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. Correspondingly, the normalized system throughput is:

$$\eta_{\text{mS-Aloha-uw}}(0 \le k \le 0.5) = \lambda T_t (P'_{S_1} + P'_{S_2} + P'_{S_3})$$
 (23)

Case 2: $0.5 \le k \le 1.0$

The frame success probability in this case is given by:

$$P_S(0.5 \le k \le 1.0) \stackrel{\Delta}{=} P_{s_1}'' + P_{s_2}'' + P_{s_3}''$$
(24)

Using (10) and (13) we have,

$$P_{s_1}^{\prime\prime} = \frac{e^{-2\lambda T_{sk}}}{\lambda T_{sk}} \left[e^{\lambda T_{sk}} - e^{\lambda T_{sk}k^2} \right] - \frac{2ke^{-\lambda T_{sk}}}{\sqrt{\lambda T_{sk}}} \left[D_+ \left(\sqrt{\lambda T_{sk}}\right) - e^{-\lambda T_{sk}(1-k^2)} D_+ \left(k\sqrt{\lambda T_{sk}}\right) \right].$$
(25)

$$P_{s_2}'' = \int_{r=R-kR}^{kR} e^{-\lambda T_{sk}} \cdot \frac{2rdr}{R^2} = e^{-\lambda T_{sk}} (2k-1).$$
(26)

$$P_{s_3}^{\prime\prime} = e^{-\lambda T_{sk}} \left[\frac{1 - e^{-\lambda T_{sk}(1-k)^2}}{\lambda T_{sk}} + k \sqrt{\frac{\pi}{\lambda T_{sk}}} \operatorname{erf}\left(\sqrt{\lambda T_{sk}}(1-k)\right) \right].$$
(27)

The corresponding normalized system throughput is:

 $\eta_{\text{mS-Aloha-uw}}(0.5 \le k \le 1.0) = \lambda T_t \cdot (P_{s_1}'' + P_{s_2}'' + P_{s_3}'')$ (28)

VI. RESULTS AND DISCUSSION

System throughput performance have been studied in MAT-LAB using the analytic expressions developed in Sections III, IV, and V, and via C based discrete event simulations.

The channel rate was taken $R_c = 16$ kbps. Default and the largest frame size F were 40 Bytes and 240 Bytes, respectively. Default value of R was 20 m. $T_t = \frac{F}{R_c}$. Acoustic signal speed is v = 1500 m/s.

In simulations, N = 200 randomly located nodes were taken around a receiver's communication range. In each iteration, a randomly located transmitter was chosen, and the other neighboring transmitters' activities were controlled by varying the frame arrival rate. Sufficient iterations were taken to obtain a high confidence over the simulated data.

Due to space constraint, we show only the important results. In Fig. 4 note that, the maximum throughput is monotonically decreasing as T_p^{\max} increases. This observation prompts us to restrict our mS-Aloha-uw studies to $\frac{T_p^{\text{max}}}{T_t} \leq 1$, beyond which the performance of simple Aloha will be better. The



rate of decay is not sharp after $\frac{T_p^{\max}}{T_t} = 1$, which is because, beyond this value there is a finite probability of receiving a frame correctly even though there could be more than one transmissions within the coverage range of a receiver.

Fig. 5 indicate that, by properly choosing the slot size UW S-Aloha performance can be highly improved. Note that k = 0 implies the slot size $T_{sk} = T_t$, and it gives the same throughput performance as in Aloha. A good match of the analytic and



simulation results also verify correctness of the analysis.

Finally, in Fig. 6, on the Y1 axis the optimum slot size reduction factor k that achieves $\eta_{\text{mS-Aloha-uw}}^{\text{max}}$ is plotted with respect to $\frac{T_p^{\text{max}}}{T_t}$, which can be controlled either by varying R or T_t . In conjunction, the percentage throughput gain with



Fig. 6. mS-Aloha-uw performance gain over naive S-Aloha-uw.

respect to the naive S-Aloha-uw at the k_{opt} values, which is

defined as:

$$Gain = \frac{\eta_{mS-Aloha-uw}^{max} - \eta_{S-Aloha-uw}^{max}}{\eta_{S-Aloha-uw}^{max}} \times 100, \qquad (29)$$

is also plotted on the Y2 axis. It shows the maximum gain of mS-Aloha-uw over naive S-Aloha-uw is 17.3%, at $T_p^{\max} = T_t$. The plots further demonstrate that, while naive S-Aloha-uw does not offer system throughput as good as in S-Aloha-rf, an optimal choice of slot size can offer an appreciable increase in throughput, especially at large nodal coverage range.

VII. CONCLUSION

In this paper, we have presented a theoretical framework for throughput computation in underwater acoustic wireless networks. We have shown that Aloha performance does not have any impact, while slotted Aloha does have a strong impact, of signal propagation speed. Further, we have proposed a new aggressive slotting concept, wherein the slot size can be optimally chosen such that, even by allowing some collisions due to overshooting the slot boundary, the overall system throughput can be significantly increased. Our analytic conclusions have been verified by discrete event simulations.

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