A Novel Hedonic Coalition Formation Game for Spectrum Shared Communication in CBRS Band

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***Abstract*—Citizen Broadband Radio Service (CBRS) uses the spectrum of the 3.5 GHz band. It allows general authorized access (GAA) users to access the spectrum through the spectrum access system (SAS) while ensuring no interference with incumbent users and priority access license (PAL) holders. This work proposes a novel channel allocation scheme for GAA users, implemented through the SAS. The proposed schemes are based on a cooperative game-theoretic approach called the hedonic coalition formation (HCF) game and genetic algorithm (GA). In the former, each coalition represents a group of users sharing the same channel within the 3.5 GHz band, whereas the coalitions are formed to minimize cochannel interference (CCI). In the latter, GA is utilized to find a suboptimal channel allocation. This work demonstrates the effectiveness of the proposed schemes in comparison with Approach 1 in the standard.**

***Index Terms*—Citizen Broadband Radio Service, Binary quadratic programming, Coalition formation game.**

I. Introduction

The 5G New Radio (NR) supports a broad spectrum of operating frequencies, including low-band (700 MHz), mid-band (3.4-3.8 GHz), and millimeter-wave bands (26 GHz) [1]. Such a wide range of operating frequencies makes 5G NR versatile, making it well-suited for diverse use cases and deployment scenarios. It was primarily planned for the mid-band, but the growing demand for higher data rates and lower latency exacerbates the strain on the available spectrum. Meanwhile, the Citizens Broadband Radio Service (CBRS) in the United States, where the 3.5 GHz band has been allocated to the Department of Defense (DoD), represents a pioneering approach to sharing the 3.5 GHz band, which offers a potential pathway to meet the growing needs of 5G networks.

The CBRS operates on a three-tiered access model: Incumbent Access, Priority Access License (PAL), and General Authorized Access (GAA), whereas the 3.5 GHz band is divided into 15 orthogonal 10 MHz channels. Incumbent users, such as Navy shipborne, are protected through dynamic protection areas (DPA); PAL users acquire licenses via competitive bidding and are assigned specific channels by the Spectrum Access System (SAS), enjoying interference protection within PAL protection areas (PPAs). GAA users, however, opportunistically access the remaining spectrum without interference protection and must avoid causing harmful interference to PAL and incumbent users. This creates significant challenges in ensuring efficient coexistence among GAA users and with higher-tier users.

To address GAA coexistence with the incumbent and PAL users, the Wireless Innovation Forum (WInnForum) proposed three approaches: Approach 1, 2, and 3 [4]–[6]. These methods primarily rely on graph coloring, where GAA users are represented as vertices or nodes, and edges indicate interference conflicts. Approach 1 focuses solely on channel allocation, while Approach 2 jointly considers channel allocation and power control. Approach 3 extends this further by maximizing channel utilization through recursive clustering. While these methods are operationally simple and effective for moderate deployments, they are primarily designed for maximizing channel utilization and provide limited optimization for co-channel interference (CCI) among GAA users.

For prior work, the performance of Approach 1 has been examined in [7]. As in Approach 2, joint channel and transmit power allocation schemes are considered in [8], [9]. Graph coloring method is used in [8], whereas mixed-integer linear programming (MILP) is utilized [9]. In contrast with [7]–[9], where channel allocations only for GAA users are considered, joint channel allocation schemes for PAL holders and GAA users are considered in [10], [11], both aiming to maximize channel utility. The optimization problem in [10] is formulated as a mixed-integer nonlinear program (MINLP) with interference protection for PAL holders as a constraint. Ying *et. al* [11] proposed a heuristic greedy algorithm for graph coloring to maximize channel allocation. Apart from traditional algorithms, Q-learning has enabled GAA users to bid for PAL idle channels, aiming to enhance channel utilization [12]. Additionally, a game-theoretic distributed channel allocation algorithm is considered for the TV white space spectrum [13], where a database server with geo-location information on access points (APs) works as the SAS.

We consider centralized channel allocation algorithms for the SAS to employ in the CBRS, focusing on minimizing CCI among GAA users. The main contributions are as follows:

- We model the channel allocation problem as a constrained binary quadratic programming (BQP) problem and show that its complexity is NP-hard.
- From a game-theoretic perspective, we formulate the channel selection as a hedonic coalition formation (HCF)

TABLE I
NOTATION AND DEFINITION IN SYSTEM MODEL

Notation	Definition
$\mathcal{N}$	Set of indices of N GAA users
$\mathcal{K}$	Set of indices of K available GAA channels
$\mathcal{L}$	Set of indices of L contiguous channel allocation patterns
C	Number of contiguous channels used by each GAA user
C_k	CCI of channel k
$\overline{C}_l$	CCI of channel pattern l
$x_{k,n}$	A binary variable indicating whether channel k is used (1) or not used (0) by GAA user n
$y_{n,l}$	A binary variable indicating whether contiguous channel allocation pattern l is used (1) or not used (0) by GAA user n
$\alpha_{i,j}$	Long-term average received power from GAA user j to user i on the same channel
Ψ	$N \times N$ co-channel interference power matrix among users
$\mathcal{T}_j$	Equivalent isotropically radiated power (EIRP) of GAA user j
$\gamma_{i,j}$	Pathloss from GAA user j to user i
P	$L \times K$ matrix representing the mapping between channel indices and contiguous channel allocation pattern indices
$\mathcal{P}$	Block diagonal matrix of size N blocks, each containing P
Q	Block diagonal matrix of size K blocks, each containing Ψ

game and propose an HCF algorithm executed by the SAS to minimize CCI. The resulting channel allocations are characterized by Nash-stable and permutation-stable equilibria.

- We also propose a Genetic algorithm (GA) for channel allocation and compare the performance of the HCF algorithm and GA algorithm against Approach 1 given in [4]. Extensive simulations demonstrate the high efficiency of the proposed algorithms in minimizing CCI.

II. Channel allocation for GAA users

Section II-A presents the system model for GAA users in CBRS, whereas Section II-B introduces our objective function aimed at minimizing the overall system CCI as BQP and demonstrates that the problem is NP-hard.

A. System Model

Let a set of GAA users be denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$. They are randomly distributed within a licensed area managed by a SAS. The CBRS system has K orthogonal 10 MHz channels for GAA users, indexed as $\mathcal{K} = \{1, 2, \ldots, K\}$. As shown in Fig. 1, each GAA user takes a channel pattern consisting of C (Maximum 4 in CBRS band) consecutive 10 MHz channels. and we denote the number of channel patterns as L, with $\mathcal{L} = \{1, 2, ..., L\}$ representing the index set of channel patterns. The channels used by different channel patterns are orthogonal, with channel pattern l utilizing channels $C(l-1)+1$ to Cl. Fig. 1 illustrates the channel usage of different channel patterns for $C = 2$ and $K = 8$. The number of channel patterns L is given by $L = \left\lfloor \frac{K}{C} \right\rfloor$, where $\lfloor x \rfloor$ denotes the ceiling function that takes x to the largest integer smaller than or equal to x.

When a GAA user joins the CBRS system, it sends the SAS a registration request. This request includes detailed information for the GAA user, such as precise geographical

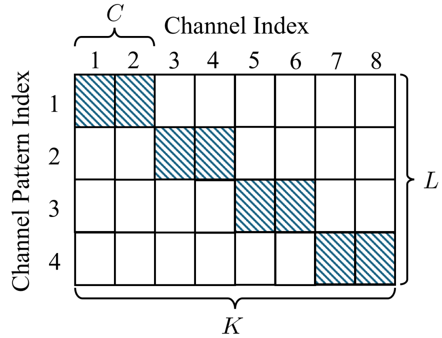


Fig. 1. The mapping of channel pattern to channels in the case of $K = 8$, $C = 2$, and $L = 4$.

coordinates, antenna height (relative to the ground), maximum transmit power, and other necessary operational parameters to meet authentication and compliance requirements [14]. Upon successful registration, the GAA user submits a channel request. The SAS applies radio propagation models to predict propagation characteristics and computes the interference levels among GAA users based on the provided data [4]. Channels are allocated to GAA users in a manner that minimizes CCI. The assigned channels typically remain in use for several days without the need to recalculate interference levels frequently. However, the SAS continuously monitors the system to accommodate GAA user registrations and deregistrations.

Let ψ be the interference matrix that the SAS constructs for channel allocation:

$$\psi = \begin{pmatrix} 0 & \alpha_{1,2} & \alpha_{1,3} & \cdots & \alpha_{1,N} \\ \alpha_{2,1} & 0 & \alpha_{2,3} & \cdots & \alpha_{2,N} \\ \alpha_{3,1} & \alpha_{3,2} & 0 & \cdots & \alpha_{3,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{N,1} & \alpha_{N,2} & \alpha_{N,3} & \cdots & 0 \end{pmatrix}, \quad (1)$$

where $\alpha_{i,j}$ is the average received power at GAA user i when GAA user j uses transmit power $\mathcal{T}_j$ on the same channel. Self-interference is neglected, i.e., $\alpha_{i,i} = 0$ for $i \in \mathcal{N}$. Assuming that the short-term fading is averaged out, $\alpha_{i,j}$ is expressed as $\alpha_{i,j} = \gamma_{i,j}\mathcal{T}_j$. Note that pathloss $\gamma_{i,j}$ is assumed to be channel-independent. For simplicity, we assume symmetric pathloss, i.e., $\gamma_{i,j} = \gamma_{j,i}$ for $i, j \in \mathcal{N}$. In the following section, the SAS calculates the CCI of GAA users using (1). In Section V-B, we describe the pathloss model for $\gamma_{i,j}$.

B. Binary Quadratic Programming for Channel Allocation

Let $\boldsymbol{y}_n = [y_{n,1}, \ldots, y_{n,L}]$ for $y_{n,l} \in \{0, 1\}$ denote a row vector showing the channel pattern that GAA user n selects. Since each GAA user takes only one channel pattern out of L patterns, it follows that for $n \in \mathcal{N}$,

$$\sum_{l \in \mathcal{L}} y_{n,l} = 1. \quad (2)$$

We define a binary row vector $\boldsymbol{y} = [\boldsymbol{y}_1\ \boldsymbol{y}_2\ ...\ \boldsymbol{y}_N]$ of length LN to indicate the channel pattern selections by GAA users. Then, we can rewrite (2) as a matrix form:

$$A\boldsymbol{y}^T = \mathbf{1}_{N \times 1}, \quad (3)$$

where $A = [\mathcal{A}_1 \ \mathcal{A}_2 \ \dots \ \mathcal{A}_N]$, each matrix $\mathcal{A}_i$ is an $N \times L$ matrix where the i-th row is filled with 1, and all other elements are 0. Thus, the size of matrix A is $N \times LN$.

On the other hand, let $\hat{x}_k$ be a column vector of length N:

$$\hat{x}_k^T = [x_{k,1} \dots x_{k,N}], \tag{4}$$

where $x_{k,n} \in \{0,1\}$ shows the k-th 10 MHz channel selected by GAA user n. When GAA user n occupies 10 MHz channel k, then $x_{k,n} = 1$; otherwise, $x_{k,n} = 0$. Let $\boldsymbol{x}$ be a column vector composed of $\hat{x}_k$ for $k \in \mathcal{K}$:

$$\boldsymbol{x}^T = [\hat{x}_1 \dots, \hat{x}_K], \tag{5}$$

which has a total length of NK. While the row vector $\boldsymbol{y}$ shows the occupancy of channel patterns, the column vector $\boldsymbol{x}$ expresses the occupancy of each channel by GAA users. We can relate $\boldsymbol{y}$ to $\boldsymbol{x}$ as follows: First, we define a channel pattern matrix P of size $L \times K$ such that an element of 1 (0) at the l-th row and k-th column can indicate that the l-th channel pattern does (does not) include channel k. Next, we define matrix $\mathcal{P}$ that takes N copies of matrix P on its main diagonal as

$$\mathcal{P} = \text{diag}(P) = \begin{bmatrix} P & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & P & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & P & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & P \end{bmatrix}, \tag{6}$$

where $\mathbf{0}$ is a zero matrix of the same size as P. Note that the size of matrix $\mathcal{P}$ is $NL \times NK$. To build the correspondence between the elements of $\boldsymbol{x}$ and $\boldsymbol{y}$, we introduce a square matrix $\mathcal{V}$ of size $KN \times KN$, which consists of block matrix

$$\mathcal{V} = \begin{bmatrix} V_{11} & V_{12} & \dots & V_{1K} \\ V_{21} & V_{22} & \dots & V_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ V_{N1} & V_{N2} & \dots & V_{NK} \end{bmatrix}, \tag{7}$$

where matrix V_{ij} of size $K \times N$ consists of all zeros, except for the j-th row and i-th column element v_{ji} equal to one. Then, the column vector $\boldsymbol{x}$ can be expressed in terms of $\boldsymbol{y}$ as

$$\boldsymbol{x}^T = \boldsymbol{y}\mathcal{P}\mathcal{V}. \tag{8}$$

Let us show how to get the CCI of channel k using column vector $\boldsymbol{x}$. Let C_k for $k \in \mathcal{K}$ be the CCI of channel k, which can be expressed as

$$C_k = \frac{1}{2}\hat{x}_k^T \psi \hat{x}_k, \tag{9}$$

where interference matrix ψ is defined in (1). By summing up C_k for $k \in \mathcal{K}$, we get the overall CCI:

$$\frac{1}{2}\sum_{k \in \mathcal{K}} C_k = \frac{1}{2}\boldsymbol{x}^T \Psi \boldsymbol{x}, \tag{10}$$

where matrix Ψ consists of K copies of matrix P on its main diagonal, i.e., $\Psi = \text{diag}(\psi)$. Since we can write the column vector $\boldsymbol{x}$ using the row vector $\boldsymbol{y}$ in (8), in order to express the CCI with $\boldsymbol{y}$, let us define matrix Q as

$$Q = \mathcal{P}\mathcal{V} \cdot \Psi \cdot \mathcal{V}^T\mathcal{P}^T. \tag{11}$$

The channel pattern allocation problem to minimize the CCI is formulated as a BQP:

$$\begin{aligned} \underset{\boldsymbol{y}}{\text{minimize}} \quad & \frac{1}{2}\boldsymbol{y}Q\boldsymbol{y}^T \\ \text{subject to} \quad & A\boldsymbol{y}^T = \mathbf{1}_{N\times 1}, \\ & y_{n,\ell} \in \{0,1\}, \quad n \in \mathcal{N}, \ \ell \in \mathcal{L}. \end{aligned} \tag{12}$$

The following theorem characterizes the BQP.

Theorem 1. *The BQP with the linear equality constraint in* (12) *is a nonconvex and NP-hard problem.*

Proof: Based on (8) and (11), it can be seen that the objective function in (12) is equivalent to $\frac{1}{2}\boldsymbol{x}^T\Psi\boldsymbol{x}$, where $x_{k,n} \in \{0,1\}$. The matrix Ψ is symmetric and has a rank of NK, implying that it has NK real eigenvalues. Since all the diagonal elements of Ψ are zero, the trace of Ψ is 0, which means that the sum of its eigenvalues is zero. Consequently, some (real) eigenvalues of Ψ are positive, while others are negative. This implies that Ψ is an indefinite matrix, leading to the non-convexity of the QP. As proven in [17], since the matrix Ψ in the QP has at least one negative eigenvalue, the problem is NP-hard. Therefore, even without considering the constraints, the original problem (12) remains NP-hard. The constraints are linear and do not reduce the problem's complexity [18], which confirms that the BQP in (12) is NP-hard. ■

III. Cooperative Game Approach

Section III-A addresses the optimization problem of (12) through an HCF game, and Section III-B proposes the HCF algorithm that SAS utilizes to allocate channel patterns for GAA users.

A. Hedonic Coalition Formation Game

Suppose N GAA users, each of whom can be indexed by a set $\mathcal{N}$. Hereafter, these users will be referred to as players. A set of GAA users who select channel pattern l is denoted by $\mathcal{H}_l$ for $l \in \mathcal{L}$. It is a non-empty subset of $\mathcal{N}$, which we call *coalition* $\mathcal{H}_l$. A coalition structure is a partition Π of the set of GAA users, i.e., $\mathcal{N}$ into disjoint coalitions; that is, $\Pi = \{\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_L\}$. Given a coalition structure Π and a player i, let $\mathcal{H}_\Pi(i)$ denote the set $\mathcal{H}_l \in \Pi$ such that GAA i belongs to coalition $\mathcal{H}_l$, i.e., $i \in \mathcal{H}_l$. Each player $i \in \mathcal{N}$ has a preference relation represented by an order $\succeq_i$ (a reflexive, complete, and transitive binary relation) over the set $\{\mathcal{H}_l \subseteq \mathcal{N} : i \in \mathcal{H}_l\}$. An HCF game is defined as $G_H = (\mathcal{N}, (\succeq_i)_{\forall i \in \mathcal{N}})$, and if the following two conditions are met:

1) The payoff of any player depends only on the members of the coalition to which the player belongs
2) The coalitions form as a result of the preferences of the players over the possible coalition's set

To formulate the channel allocation for GAA users as an HCF game, we write the CCI of channel pattern l as the cost of coalition $\mathcal{H}_l$:

$$\overline{C}_l = \frac{1}{2} \sum_{i \in \mathcal{H}_l} c_i(\mathcal{H}_l), \tag{13}$$

where $c_i(\mathcal{H}_l)$ represents the sum of the CCI costs for player i occupying channel pattern l of coalition $\mathcal{H}_l$. We have $c_i(\mathcal{H}_l)$ as

$$c_i(\mathcal{H}_l) = y_{i,l} \sum_{n=1}^{N} \sum_{j=1}^{L} q_{L(i-1)+l, L(n-1)+j} y_{n,j}. \tag{14}$$

where $q_{i,j}$ is the i-th row and j-th column element of Q in (11). In (14), the CCI that GAA user i receives with channel pattern l is found by the inner product of the $L(i-1)+l$-th row of matrix Q and vector $\boldsymbol{y}^T$. It is notable that C_k in (9) shows the CCI of channel k for $k \in \mathcal{K}$, whereas $\overline{C}_l$ in (14) indicates the CCI of channel pattern l for $l \in \mathcal{L}$.

In the HCF game G_H, GAA user i chooses channel pattern l, i.e., joining coalition $\mathcal{H}_l$, to minimize its cost $c_i(\mathcal{H}_l)$ over $\mathcal{H}_l$ for $l \in \mathcal{L}$, whereas the valuation on coalition l in (13) is listed in the ascending order of CCI C_l for $l \in \mathcal{L}$. For any two coalitions $\mathcal{H}_l$ and $\mathcal{H}_m$ that include player i, we define a preference relation for player i as $\mathcal{H}_l \succeq_i \mathcal{H}_m$, i.e., $\mathcal{H}_l$ is preferred to $\mathcal{H}_m$:

$$\mathcal{H}_l \succeq_i \mathcal{H}_m \Leftrightarrow c_i(\mathcal{H}_l) \leq c_i(\mathcal{H}_m). \tag{15}$$

From (15), it is shown that GAA i prefers channel pattern l to m when CCI of channel pattern l is less than that of channel pattern m. The outcome of this game is evaluated by the stability of the coalition structure. Three well-known stabilities are introduced as follows.

Definition 1. *A partition Π is said to be Nash-stable if there is no player who benefits from leaving its present coalition to join another coalition of the partition. A partition Π is Nash-stable if $\forall i \in \mathcal{N}$,*

$$\mathcal{H}_\Pi(i) \succeq_i \mathcal{H}_l \cup \{i\} \tag{16}$$

for all $\mathcal{H}_l \in \Pi \cup \{\emptyset\}$.

Definition 1 implies that the partition Π is Nash stable if no individual player i would be strictly better off by moving into a different coalition inside the same structure. In this case, a deviation of player i is allowed even if the coalition members joined by i are made worse off by the deviation.

Definition 2. *A partition Π is individually stable if there exists no player $i \in \mathcal{N}$ and no coalition $\mathcal{H}_l \in \Pi \cup \{\emptyset\}$ such that $\mathcal{H}_k \cup \{i\} \succ_i \mathcal{H}_\Pi(i)$, $\mathcal{H}_l \cup \{i\} \succeq_j \mathcal{H}_k$ for all $j \in \mathcal{H}_l$.*

Definition 2 means that the new coalition members do not accept newly joining player i because this player i would put some existing member into a worse situation. If a partition Π is Nash-stable, it is also individually stable.

Theorem 2. *Nash-stable and individual stable partitions exist for the hedonic game of CBRS channel selection.*

Algorithm 1 HCF Algorithm

1: Generate an initial (random) coalition.
2: Execute Nash-stable partition function
3: Execute Coalition exchange function

Proof: Since Nash stability implies individual stability [19], we focus on the existence of Nash stability. It was proven that for a hedonic game to have Nash stability, it should have *additively separable and symmetric preferences* (ASSP) [19], [20]: A hedonic game is said to have ASSP when each player $i \in \mathcal{N}$ has a value $v_i(j)$ for player j being in the same coalition as i, which is symmetric, i.e., $v_i(j) = v_j(i)$. In addition, for player i in coalition S, it gets utility $\sum_{j \in S \setminus \{i\}} v_i(j)$. For two coalitions T_1 and T_2 that player i chooses to join, we have $T_1 \succeq_i T_2$ if and only if $\sum_{j \in T_1 \setminus \{i\}} v_i(j) \geq \sum_{j \in T_2 \setminus \{i\}} v_i(j)$. From (13), our hedonic game G_H satisfies these properties when the cost is converted into the utility with the sign change, i.e., $v_i(j) = -c_i(\mathcal{H}_l)$; a coalition with a lower CCI becomes one with a higher utility. ∎

Definition 3. *A partition Π is permutation-stable if there does not exist a player pair $\{i,j\} \in \{\{i,j\} \mid \mathcal{H}_\Pi(i) \neq \mathcal{H}_\Pi(j)\}$ such that*

$$C_{\mathcal{H}_\Pi(i) \setminus \{i\} \cup \{j\}} + C_{\mathcal{H}_\Pi(j) \setminus \{j\} \cup \{i\}} < C_{\mathcal{H}_\Pi(i)} + C_{\mathcal{H}_\Pi(j)}. \tag{17}$$

Definition 3 indicates that if swapping players between any two different coalitions cannot reduce the total cost of the two coalitions, then the partition Π is permutation-stable.

Since Nash-stable implies individual stability, we need an efficient HCF algorithm to find Nash-stable and permutation-stable partitions.

B. Hedonic Coalition Formation Algorithm

As in Definition 3, it can be expected that performing the exchange operation on any non-permutation-stable partition to obtain a permutation-stable partition can always reduce the overall CCI. Therefore, in this section, we propose the HCF algorithm, which performs the exchange operation on a Nash-stable partition to achieve permutation stability. This algorithm outperforms one that considers only Nash stability regarding CCI performance.

Algorithm 1 shows the proposed HCF algorithm. Initially, each GAA user $i \in \mathcal{N}$ randomly joins coalition $\mathcal{H}_l$ for $l \in \mathcal{L}$, i.e., randomly selects a channel pattern. Then, two functions are executed: the *Nash-stable partition* that moves each player iteratively to achieve Nash stability, and the *Coalition Exchange*, where players from different coalitions are exchanged to achieve permutation stability further, as described in Algorithms 2 and 3.

The Nash-stable partition in Algorithm 2 works as follows: It first calculates the CCI for each coalition and sorts the coalitions in descending order of CCI to enhance the algorithm's efficiency. In lines 4 to 11, the algorithm examines each coalition, calculating the change in CCI, denoted by ΔC, for each player when considering joining another coalition.

Algorithm 2 Nash-Stable partition

1: Calculate CCI with an initial allocation, initialize $cnt = 0$ and $r = 1$
2: **while** (1) **do**
3: Sort coalitions in the descending order of CCIs & $C_{\max} = 0$
4: Find the r-th maximum CCI channel, say $\mathcal{H}_k$
5: **for** each GAA user i in $\mathcal{H}_k$ **do**
6: Calculate CCI of $\mathcal{H}_k \backslash \{i\}$: C_k°
7: **for** $l \in \mathcal{L} \backslash \{k\}$ **do** % for each coalition except i
8: Get CCI of $\mathcal{H}_l \cup \{i\}$: $C_l^\bullet$ and $\Delta C = C_k^\circ - C_l^\bullet$
9: **if** $\Delta C > C_{\max}$ **then**
10: Set $C_{\max} = \Delta C$
11: Store GAA user i and channel pattern l
12: $\mathcal{H}_l \leftarrow \mathcal{H}_l \cup \{i\}$ and $\mathcal{H}_k \leftarrow \mathcal{H}_k \backslash \{i\}$
13: **if** $C_{\max} = 0$ **then**
14: $cnt = cnt + 1$
15: **if** $cnt = K$ **then**
16: BREAK
17: $r \leftarrow r + 1$
18: **if** $r = K + 1$ **then**
19: $r = 1$
20: **else**
21: $cnt = 0$

It identifies the player and coalition that would yield the maximum ΔC, and in line 12, the player is reassigned to the selected coalition. Specifically, let C_k° denote the reduction in CCI of coalition $\mathcal{H}_k$ when GAA user i leaves coalition k in line 6:

$$C_k^\circ = \frac{1}{2} \sum_{j \in \mathcal{H}_k} c_j(\mathcal{H}_k) - \frac{1}{2} \sum_{j \in \mathcal{H}_k \backslash \{i\}} c_j(\mathcal{H}_k \backslash \{i\}). \tag{18}$$

Furthermore, $C_l^\bullet$ denotes an increase in CCI of coalition $\mathcal{H}_l$ when GAA user i would join $\mathcal{H}_l$ in line 8:

$$C_l^\bullet = \frac{1}{2} \sum_{j \in \mathcal{H}_l \cup \{i\}} c_j(\mathcal{H}_l \cup \{i\}) - \frac{1}{2} \sum_{j \in \mathcal{H}_l} c_j(\mathcal{H}_l). \tag{19}$$

The differential CCI when GAA user i moves from coalition $\mathcal{H}_k$ (resulting in a CCI decrease) to $\mathcal{H}_l$ (resulting in a CCI increase) is defined as

$$\Delta C = C_k^\circ - C_l^\bullet = c_i(\mathcal{H}_k) - c_i(\mathcal{H}_l \cup \{i\}). \tag{20}$$

Notice that ΔC represents the CCI reduction when a GAA moves from coalition $\mathcal{H}_k$ to coalition $\mathcal{H}_l$, which also reflects the reduction in overall CCI, as other coalitions remain unaffected. In lines 9 to 11, $C_{\max}$ records the largest ΔC, effectively serving as a gradient for CCI. Algorithm 2 maximizes the reduction in CCI at each iteration, enhancing its efficiency. Lines 13 to 21 check whether the partition is Nash-stable, which serves as the termination condition. To this end, we propose the following theorem.

Theorem 3. *A solution of Algorithm 2 always converges to a Nash-stable partition.*

Algorithm 3 Coalition Exchange

1: **while** (1) **do**
2: $Exchange\_rec = 0$;
3: **for** $k = 1 : \mathcal{L}$ **do**
4: Calculate the CCI of $\mathcal{H}_k$ and set it to C_{old}
5: **for** $l = 1 : \mathcal{L} \backslash \{k\}$ **do**
6: **for** each GAA user i in $\mathcal{H}_k$ and GAA user j in $\mathcal{H}_l$ **do**
7: Exchange two GAA users i and j in each coalition
8: Find CCI with this exchange: C_{new}
9: **if** $C_{\text{new}} < C_{\text{old}}$ **then**
10: $C_{\text{old}} = C_{\text{new}}$ and record this exchange $Exchange\_rec = Exchange\_rec + 1$
11: Execute the exchange in line 8 and set $C_{\text{old}} = C_{\text{new}}$
12: **if** $Exchange\_rec == 0$ **then**
13: BREAK

Proof: In Algorithm 2, according to the conditional statement in line 9, $C_{\max} = 0$ if and only if $\Delta C \leq 0$ throughout the loop in line 5. From (20), for each $i \in \mathcal{H}_k$ and $l \in \mathcal{L} \setminus \{k\}$ we have

$$c_i(\mathcal{H}_k) \leq c_i(\mathcal{H}_l \cup \{i\}) \Rightarrow \mathcal{H}_k \succeq_i \mathcal{H}_l \cup \{i\}. \tag{21}$$

This means that none of the GAA users in coalition $\mathcal{H}_k$ can reduce the CCI by switching to another coalition. Therefore, the condition in line 13 being true indicates that GAA users in the current coalition have no incentive to leave. We refer to such a coalition as a "no-incentive" coalition). At this point, the algorithm takes two actions: incrementing cnt by 1 to record the consecutive occurrence of no-incentive coalitions and incrementing r by 1 to proceed to the next coalition for the GAA user in lines 5 to 12. The algorithm terminates at line 16 when $cnt = K$. This indicates that all K coalitions are "no-incentive" coalitions, meaning that no users can find a more preferred coalition, as shown in (21). This satisfies the conditions of Definition 1, and confirms that the partition Π produced by Algorithm 2 is Nash-stable. ■

Let us consider the coalition exchange in Algorithm 3: It calculates the change in CCI if GAA user i in coalition $\mathcal{H}_k$ would be exchanged with GAA user j in coalition $\mathcal{H}_l$. It exchanges two GAA users in two different coalitions if exchanging them yields CCI (C_{new}) less than the current CCI (C_{old}). When the condition in line 12 is satisfied, it indicates that no better exchange can be made, and thus permutation-stability is achieved. When the initial partition is made into a Nash stable partition, the coalition exchange routine can further reduce the overall CCI. However, it is worth noting that the coalition exchange may disrupt a Nash stable partition, as it focuses only on minimizing the total cost of the two coalitions without considering whether individual players within each coalition remain satisfied

IV. The GA for Channel Pattern Allocation

In this section, we propose a GA to find a solution for (12). GAs initially choose a set of random candidate solutions. These candidate solutions evolve over several generations through crossover and mutations such that more fitted candidates close to the best or near-optimal solution to a given problem can be produced and survive.

The GA algorithm utilizes a population $\mathcal{G}$ of M individuals as a potential solution to the channel pattern allocation problem defined in (12). Potential solutions undergo genetic operations such as crossover and mutation, which exchange and modify the elements among individuals to produce a better potential solution. As this is iteratively done, the GA explores the solution space of (12) and forms a new and better population from the previous one. Let $\boldsymbol{z}^{(m)}$ for $m \in \{1, \ldots, M\}$ be one individual, which is a vector of length N: $\boldsymbol{z}^{(m)} = [z_n^{(m)}]$, where $z_n^{(m)} \in \mathcal{L}$ for $n \in \mathcal{N}$ is the n-th element of $\boldsymbol{z}^{(m)}$, indicating the channel allocation pattern employed by GAA n. Let $y_{n,l}^{(m)}$ be the m-th candidate solution for $y_{n,l}$ in (12). As $z_n^{(m)}$ takes an integer value, we can relate it to $y_{n,l}^{(m)}$ as

$$y_{n,l}^{(m)} = \begin{cases} 1, & \text{if } l = z_n^{(m)} \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

Let us introduce the key features of the GA, such as fitness function, tournament selection, crossover, and mutation. First, the fitness function f serves as the criterion for evaluating the quality of individuals. Since our objective is to minimize the overall CCI, the fitness function f is naturally adopted as (12):

$$f(\boldsymbol{y}^{(m)}) = \frac{1}{2}\boldsymbol{y}^{(m)} Q [\boldsymbol{y}^{(m)}]^T. \tag{23}$$

A lower value of the fitness function indicates a better individual. Second, the tournament selection in line 9 chooses superior individuals and generates a new population $\mathcal{G}$. The function Tournament_selection($\mathcal{G}$, T_s) chooses T_s random individuals from the original population $\mathcal{G}$ and picks up the best individual among them chosen as a new individual. This is repeated until a new population of M individuals is formed. Third, the crossover operation combines the superior traits of the new population $\mathcal{G}$. It picks up a pair of individuals randomly, say a and b. For a crossover point randomly selected from $\{1, \ldots, N-1\}$, say k, the function Crossover($\boldsymbol{z}^{(a)}$, $\boldsymbol{z}^{(b)}$) combines them as: $\boldsymbol{z}^{(a)} = \left[z_1^{(a)}, \ldots, z_k^{(a)}, z_{k+1}^{(b)}, \ldots, z_N^{(b)}\right]$, $\boldsymbol{z}^{(b)} = \left[z_1^{(b)}, \ldots, z_k^{(b)}, z_{k+1}^{(a)}, \ldots, z_N^{(a)}\right]$. Finally, the mutation increases population diversity by randomly altering a single element. The function Mutation($\boldsymbol{z}^{(m)}$, p_m) mutates $\boldsymbol{z}^{(m)}$ as

$$z_n^{(m)} = \begin{cases} z_n^{\text{new}}, & \text{if rand()} < p_{\text{m}}, \\ z_n^{(m)}, & \text{otherwise,} \end{cases} \tag{24}$$

where z_n^{new} represents a random value within $\mathcal{L}$, and p_{m} denotes the mutation probability. The function rand() generates a random number in the unit interval.

Algorithm 4 demonstrates the process of the GA, where the population $\mathcal{G}$ iterates under the aforementioned operations until the maximum number of iterations max_iter is reached.

Algorithm 4 Centralized Genetic Algorithm

```
1:  Set M = 50, iter = 0, max_iter = 1000, T_s = 2,
    p_m = 0.01.
2:  Initialize population G = {z^(1), z^(2), ..., z^(M)}
3:  while iter < max_iter do
4:      iter ← iter + 1
5:      for m = 1 to M do
6:          Derive y^(m) based on z^(m) using (22)
7:          Compute fitness value f(y^(m)) using (23)
        /* Tournament selection */
8:      for m = 1 to M do
9:          z^(m) = Tournament_selection(G, T_s)
        /* Crossover and mutation procedures */
10:     for i = 1 to M/2 do
11:         Pick two individuals z^(a) and z^(b) randomly
12:         {z^(a), z^(b)} = Crossover(z^(a), z^(b))
13:     for m = 1 to M do
14:         z^(m) = Mutation(z^(m), p_m)
15: Select the best individual z* from the final population G
```

V. Numerical Results

In this section, we evaluate the performance of the proposed HCF and GA algorithms in terms of overall CCI as the primary performance metric. We compare them against the WInnForum Approach 1 [4] serving as the baseline because the proposed two algorithms and Approach 1 consider the channel allocation only, without transmit power control. All three algorithms operate as centralized solutions executed by the SAS.

A. Approach 1

Approach 1 assumes that GAA users as nodes in a graph with an interference threshold [4]. An *edge* is added between any two nodes if the interference power between them exceeds the threshold in either direction. This forms an interference graph. Graph coloring is used to determine the chromatic number χ, which is the minimum number of colors required so that no adjacent nodes share the same color. The total bandwidth B is equally divided among these colors to allocate channels. The bandwidth of each color i, denoted by B_i, is determined by $B_i = B/\chi$. If each B_i becomes non-integer valued MHz frequency bands, the interference threshold θ can be adjusted to achieve integer-valued bands.

B. Pathloss Model

We randomly deploy N GAA users within an area of 700 m^2 and configure their indoor/outdoor status, height, and other parameters according to R2-SGN-04 requirements in the WInnForum standard [15], including the propagation model. The parameter settings are given in Table II. For two GAA users, say i and j, who are separated by distance d, and are equipped with antenna of heights h_i and h_j each, $\gamma_{i,j}$ is expressed as

$$\gamma_{i,j} = \begin{cases} L_1(f, d, h_i, h_j), & \text{for } d \leq 0.1\text{km}, \\ L_2(f, d, h_i, h_j), & \text{for } 0.1 \leq d \leq 1\text{km}, \end{cases} \tag{25}$$

TABLE II
SIMULATION PARAMETERS FOR DENSE URBAN SCENARIO

GAA User Parameters	**Value**
Indoor-Outdoor Ratio	80% Indoor, 20% Outdoor
EIRP - Outdoor	23 dBm/10 MHz
EIRP - Indoor	20 dBm/10 MHz
Antenna Height - Outdoor	20 m
Antenna Height - Indoor	75%: 20 to 30 m, 25%: 33 to 60 m
Propagation Settings	**Value**
Frequency f	3625 MHz
Loss for $d \leq 0.1$ km	Refer to Eq. (26)
Loss for $0.1 < d \leq 1.0$ km	Refer to Eq. (27)
Building Loss	15 dB

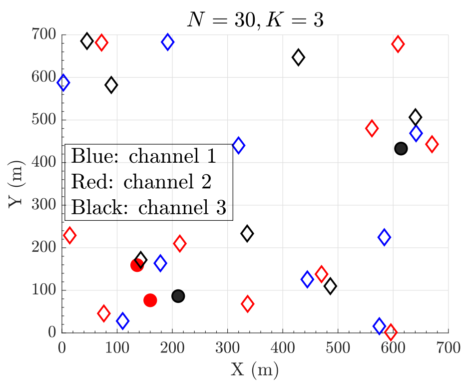


Fig. 2. GAA locations and channel selection under HCF algorithm (Diamonds for indoor, circles for outdoor).

where $L_1(f, d, h_i, h_j)$ represents free space propagation loss:

$$L_1(f, d, h_i, h_j) = 20\log_{10}(R(d, h_i, h_j)) + 20\log_{10}(f) - 27.56, \quad (26)$$

with $R(d, h_i, h_j)$ given by $R(d, h_i, h_j) = ((1000d)^2 + (h_j - h_i)^2)^{\frac{1}{2}}$. In (25), for $d \geq 1$ km, $L_2(f, d, h_i, h_j)$ incorporates the Extended Hata model in [16] as follows:

$$\begin{aligned} L_2(f, d, h_i, h_j) =& L_1(f, 0.1, h_i, h_j) + [1 + \log_{10}(d)] \times \\ & [L_{eh}(f, h_j, h_i, 1 \text{ km}) - L_{fs}(f, 0.1)], \end{aligned} \quad (27)$$

where $L_{eh}(f, h_j, h_i, d)$ denotes the Extended Hata model, defined as:

$$\begin{aligned} L_{eh}(f, h_j, h_i, d) =& 97.62 + 3.19\log_{10} f + 4.45(\log_{10} f)^2 \\ & - 13.82\log_{10} h_j - 3.2(\log_{10}(11.75h_i))^2 \\ & + 4.97 + (44.9 - 6.55\log_{10} h_j)\log_{10} d. \end{aligned} \quad (28)$$

To ensure symmetric pathloss, the maximum value of the bidirectional pathloss between GAA users i and j is taken as $\gamma_{i,j}$.

C. Performance of Channel Allocation Algorithms

Fig. 2 shows the simulation result of the HCF algorithm, where three channels are allocated to 30 GAA users randomly located. Four GAA users, represented by circles, are situated indoors, while the remaining users, depicted as diamonds, are located outdoors. It highlights how GAA users are allocated to distinct channels; when more than three GAA users are located closely, distinct channels are allocated to them.

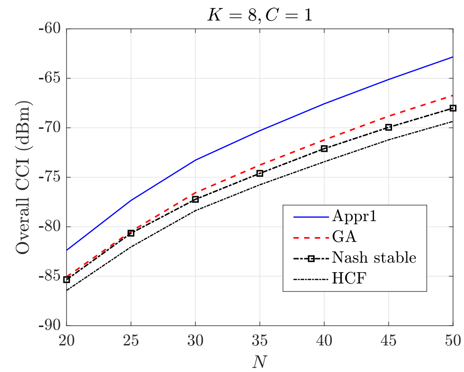
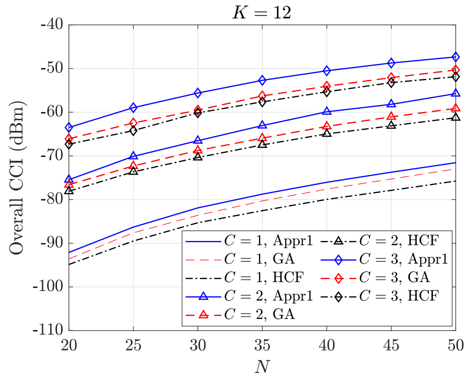


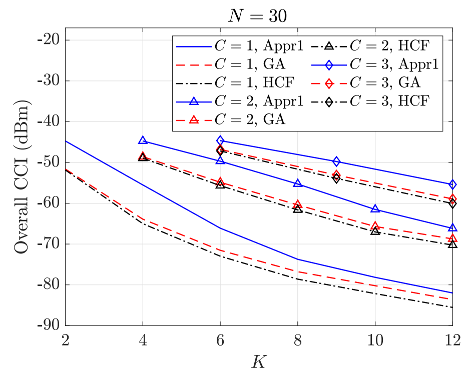
Fig. 3. Overall CCI performance over N.

Fig. 3 illustrates the CCI performance of three algorithms with $K = 8$ and $C = 1$ (single-channel allocation) as the number of GAA users N varies. In particular, we present the results by comparison of the Nash stable partition and HCF (Nash stable partition with permutation stability). It can be observed that the proposed HCF and GA algorithms achieve significantly lower CCI compared to Approach 1. Among these, the HCF algorithm demonstrates the best performance, with its relative improvement over Approach 1 becoming more pronounced as N increases. For instance, at $N = 50$, the CCI reduction reaches approximately 7 dB. The GA algorithm also outperforms Approach 1 but is inferior to the HCF algorithm, which suggests that the operations within GA, such as crossover and mutation, struggle to effectively explore the solution space of the channel allocation problem. For instance, the mutation operation causes a GAA user to switch to a different channel. This can help potential solutions move out of local minima. However, in the later stages of algorithm iterations, it often fails to reduce CCI further, leading to additional interference from the users already occupying that channel. In contrast, the HCF algorithm leverages the exchange attempts to search for better channel allocations, enabling it to achieve superior performance. It is also notable that our presentation of CCI can be a conservative measure since we assume that all GAA users are active.

In Fig. 4, we focus on the CCI performance of the HCF algorithm and GA in comparison with Approach 1, where CCI is observed as either N or K varies. We also vary C, i.e., the number of 10 MHz channels for one channel pattern. For ease of comparison across different values of C, we set $K = 12$ in Fig. 4(a). It can be observed that, for different values of C, the HCF algorithm and GA outperform Approach 1. As either C or N increases, CCI also increases due to the intensified use of channels. Note that for a larger C, the bandwidth of GAA users also increases. If the threshold of CCI is given, Fig. 4(a)



(a) CCI performance with N for $K = 12$



(b) CCI performance with varying K

Fig. 4. CCI performance of the proposed algorithms and Approach 1.

suggests the maximum number of GAA users in the area. In Fig. 4(b), we observe CCI with $N = 30$, as more channels become available. Increasing K reduces CCI as expected.

VI. Conclusions

This work examined the channel allocation for GAA users in the CBRS band in terms of BQP formulation and showed the NP-hardness of the problem. We proposed two channel allocation algorithms: One algorithm is based on an HCF game, and the other one utilizes GA, both of which the SAS can execute with the information provided by GAA users upon registration. As solution concepts for the HCF game, we explored both Nash-stable and permutation-stable partitions and developed an HCF algorithm to realize the solution concept. Two proposed algorithms were compared with Approach 1, which was given in the standard. Numerical results demonstrated that both the proposed algorithms outperformed Approach 1, and the HCF algorithm with permutation stability significantly reduced CCI.

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