Real-time Transmission Control for Multichannel NOMA Random Access Systems

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***Abstract*—To improve the throughput per channel in multi-channel non-orthogonal multiple access (NOMA) random access (RA) system, users (re)transmit their packet to one of channels using transmit power control such that the receive power of the packets at the base station (BS) can be one of the predefined levels called target receive power (TRP). The BS decodes the received packets in the descending order of the TRPs at each slot using successive interference cancellation (SIC). This work proposes real-time transmission algorithm for users to (re)transmit their packet for maximization of the system throughput. To do this, the BS estimates the number of backlogged users in real-time and adjusts and broadcasts the throughput-optimal (re)transmission probability in the algorithm. We analyze the average RA delay performance of the proposed algorithm and demonstrate its performance even with time-varying traffic.**

***Index Terms*—Non-orthogonal multiple access, online control, random access procedure.**

I. Introduction

A. Motivations

Internet-of-Things (IoTs) is a network of physical devices that can be embedded in objects such as vehicles, buildings and even livestocks. It enables us to sense and control objects remotely across existing network infrastructure. As applications of IoT have expanded over a wide range of industries such as intelligent transportation, health care, manufacturing, retail, and energy industries, the required quality-of-service (QoS) of the applications becomes more stringent, even in real-time. However, increasing number of IoT devices and unexpected massive accesses due to rapid growth of IoT applications could be an impending obstacle for stringent QoS and even can clog up the entire wireless access systems.

In order to protect radio access networks from latent long-lasting congestions and to keep strict access delay requirement, more channels over the frequency band can be employed for random access (RA) channel. Moreover, in order to improve RA capacity over the frequency channel and/or slot, power domain non-orthogonal multiple access (NOMA) has been recently examined for a RA scheme [1]–[12]. It allows users to (re)transmit their packet to the same slot and frequency channel, but requires users to control their transmit power such that their receive power at the base station (BS) can be one of the predefined target receive power (TRP) levels. The BS decodes the received packets from the highest TRP to the lowest one with successive interference cancellation (SIC). More than one packets can be decoded as long as some signal-to-interference ratio (SINR) condition is satisfied, which brings a higher throughput in RA channel. However, the packets with the same TRP cannot be separated with SIC. It is thus expected that the more the employed TRP levels, the higher the achievable throughput, i.e., the number of packets successfully decoded per slot, albeit at the expense of decoding complexity.

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In deploying uplink NOMA RA systems with multichannel for IoT devices and users, the following research questions can be raised: The first important question is how many channels and TRPs are needed at minimum in order to support a finite RA delay if a certain mean packet arrival rate of IoT devices is applied to the system. This question is not separable from what retransmission probability IoT devices or users use since a low retransmission probability can randomize the accesses over a wide span of time but it also increases RA delay. Accordingly, we may ask how the system should control retransmission probability in order to keep a bound of the average RA delay when a given mean packet arrival rate is applied. Subsequently, if the number of TRP levels is raised, it is essential to see a relation between the average power consumption and RA delay. We can also explore another question such as how the system can cope with time-varying traffic to stabilize itself, or how the maximum system throughput can be achieved if the mean of packet arrival rate varies over time. These questions are the key motivations of this work.

B. Related Work

As prior work for power-domain NOMA RA systems, in [1] Liang *et al.* implicitly made use of uplink NOMA for message 3 transmission during the four-step RA procedure of Long-Term Evolution (LTE). Particularly, the TRP levels are chosen based on the distance between the user and the BS. Uniform and beta distributions of packet arrivals are taken into account. In [2] Choi examined a lower bound of throughput of multichannel NOMA RA systems with various TRP levels. Furthermore, a much more improved lower bound has been derived and compared in [3]. It can be seen in [2], [3] that analytical complexity to obtain the throughput of uplink NOMA RA systems increased prohibitively as the number of TRP levels was raised. For the systems with two and three TRP levels, Jin and Lee in [4] obtained an upper and lower bound of the throughput, as well. Compared to [1]–[4], the focus of our work is on the RA delay rather than the throughput. For a single-channel uplink NOMA RA system, using a discrete-time Markov chain, [5] examined throughput, energy efficiency and RA delay by taking into account path-loss, Rayleigh fading, and users' spatial distribution in a cell. For unmanned aerial vehicle (UAV) assisted wireless access system, an optimal backoff algorithm for users to employ was investigated in [6]. In contrast with single-channel NOMA systems [5], [6], we analyze multichannel NOMA system using a continuous-time Markov chain, i.e., M/M/1, as an approximation. In Section IV we shall present that the simulation results verify the correctness of our considered M/M/1 approximation.

On the other hand, Abbas *et al.* considered the throughput of uplink NOMA with SIC and joint-decoding scheme in [7] when users are distributed in a cell according to a Poisson point process (PPP). In [8] Tegos *et al.* considered uplink NOMA RA with SIC and joint-decoding for wireless powered sensor networks and examined the throughput with Rayleigh- and Nakagami-fading. While this work focuses on NOMA with SIC, the framework we present might be valid for any variant of uplink NOMA RA systems.

In light of applications of machine-learning algorithm to optimize transmission control, Jang *et al.* considered a deep-learning based transmit power control in order to maximize the expectation of the minimum rate of uplink NOMA in [9]. Using the uplink NOMA RA model proposed by Liang *et al.* in [1], Wang *et al.* derived access barring scheme based on particle swarm optimization (PSO) [10], in which the mean RA delay is obtained based on simulations only. Moreover, Silver *et al.* in [11] considered an access algorithm based on Q-learning for uplink NOMA RA system, whereas Zhang *et al.* in [12] also considered an access algorithm using deep reinforcement learning (RL). It is worth noting that in [11], [12] the actions for users to take are a pair of a channel (frequency, or slot) and transmit power. The use of access algorithms based on Q-learning or RL can have some drawbacks for practical use. First, they assume that a fixed number of users are saturated; that is, they have always a packet to send. Then, they interact until the algorithm converges, i.e., all the users finally find their best action. However, in practice, an arbitrary number of users join and leave the system dynamically over time. In other words, it is highly likely that users who have completed their packet transmission may not retransmit any more. Second, the convergence time of the learning algorithms can be considered as a processing delay. If the required RA delay can be shorter than the convergence time, the algorithms fail to provide QoS. Note that Q-learning and RL can be in fact regarded as solution methods for partially observable Markov decision process (POMDP).

Compared to [10]–[12], our proposed algorithm can be considered as a solution method to POMDP as well, where the observation that the system gets is the number of idle channels at each slot; that is, the number of channels, to which no users (re)transmit their packet at each slot. This is readily available at the BS. From the observation, the underlying principle of our algorithm is to estimate the state of the hidden Markov process and allocate the best action for the backlogged users in real-time. In particular, we demonstrate the performance of our proposed algorithm even with time-varying traffic, which has not been reported in the previous work.

C. Contributions and Organization

In this work we consider multichannel uplink NOMA RA systems, where users (re)transmit their packet according to the proposed algorithm. Our contributions can be summarized as follows:

- When the users use a fixed retransmission probability, this work shows that multichannel uplink NOMA RA system with Poisson packet arrivals (with any mean) is always unstable no matter how many channels the system can have.
- In order to stabilize the system and maximize the system throughput, this work proposes real-time transmission control algorithm for uplink NOMA RA system, where users are allowed to (re)transmit only when their signal-to-noise ratio (SNR) is high enough to be above some predefined threshold. We show that the performance of the system, e.g., the mean RA delay and throughput, is invariant to the threshold.
- For Poisson distributed arrival of traffic with constant mean, the average RA delay of the proposed algorithm is analyzed with an M/M/1 approximation so that it is predictable given the number of channels.
- We test the proposed algorithm with time-varying traffic scenarios and show the robustness of the proposed algorithm under such network dynamics, e.g., Poisson packet arrivals with a time-varying mean.

This work is organized as follows: Section II introduces multichannel uplink NOMA RA system and the RA procedure. The (in)stability of this system is proved in Section III-A, whereas the proposed algorithm is derived and presented in Section III-B. Numerical results are discussed in Section IV. Finally, concluding remarks are given in Section V.

II. System Model

A. Selection of TRP Levels and SIC

Suppose a time-division duplex (TDD) system, whose time axis is divided into slots of a fixed length. Each slot is divided into two parts such as one down- and one uplink (sub)slot. We assume that a BS has N independent channels and serves users.

We assume that a user with a packet to send, i.e., backlogged user, arrive at the system according to a Poisson process with mean rate λ (packets/slot), and the users can hold only one packet, i.e., no queueing for the incoming packets. Thus, a packet and a user are indistinguishable. This model is often called infinite population model [14] where the system has an infinite set of users.

Let us assume that user i chooses a channel at random. Let h_i denote the downlink channel gain of the channel that user i has chosen, and N_0 be the power spectral density of additive white noise. We assume that the channel fading gain h_i obeys an independently and identically distributed (i.i.d.) Rayleigh distribution. In other words, h_i is complex Gaussian random variable with zero mean and unit variance for all i, i.e., $h_i \sim \mathcal{CN}(0, 1)$. Moreover, let us assume block-fading channels, where h_i remains unchanged within a slot interval and varies randomly and independently slot by slot.

In addition, the instantaneous signal-to-noise ratio (SNR) is defined as $\gamma_i = \frac{|h_i|^2}{N_0}$, which is estimated by user i through observing the pilot signals transmitted by the BS.

Let P_k for $k \in \{1, 2, \ldots, L\}$ be the k-th level of TRP at the BS and $P_L < P_{L-1} < \cdots < P_1$, where L is the number of simultaneous packet transmissions that the BS can successfully decode with power-domain NOMA technique. Owing to the channel reciprocity in TDD, user i determines his transmit power $\mathcal{P}(\gamma_i)$ based on γ_i as follows:

$$\mathcal{P}(\gamma_i) = \begin{cases} \dfrac{P_k}{\gamma_i}, & \text{for } \tau_k \leq \gamma_i \leq \tau_{k-1},\ k \in \{1, \ldots, L\} \\ 0, & \text{for } \gamma_i \leq \tau_L, \end{cases} \tag{1}$$

where τ_k is an SNR threshold for potentially selecting TRP P_k and $\tau_0 = \infty$. Therefore, only when user i's instantaneous SNR falls into an interval $[\tau_k, \tau_{k-1}]$, he considers to select TRP P_k in order to save his transmit power. In addition, (1) is often called channel inversion. Over the downlink, the BS sends a broadcast message that specifies the information about TRP levels such as the values of P_k and τ_k for $k = 1, 2, ..., L$ in the system initialization procedure. Then, based on this information, each user can adjust the transmit power according to his instantaneous SNR and (1) at each slot.

Returning to (1), let us consider the probability that user i's channel is up. Owing to the assumption of i.i.d. Rayleigh distribution on h_i, γ_i follows an exponential distribution with unit mean, i.e., $\Pr[\gamma_i \leq x] = 1 - e^{-x}$, we have

$$\phi = \Pr[\gamma_i \geq \tau_L] = e^{-\tau_L}. \tag{2}$$

Note that user i would not transmit with probability $1 - e^{-\tau_L}$ which is the probability that its SNR is below τ_L.

Suppose that j users (re)transmit at the same uplink slot. In power-domain NOMA system, the BS decodes a packet with the highest TRP first by treating other packets as interference. It then tries to decode the second highest one by subtracting the packet with the highest TRP from the received packets using SIC. Let us assume that the TRP levels that j users choose are all different. Then, user i's packet for $i \in \{1, \ldots, j\}$ and $j \in \{1, \ldots, L\}$ can be successfully decoded if its signal-to-noise interference ratio (SINR) satisfies

$$\frac{P_i}{\sum_{k=i+1}^{j} P_k + N_0} \geq \overline{\gamma}, \tag{3}$$

where $\overline{\gamma}$ denotes the decoding threshold. Note that in (3), the packets with TRP P_k for $k \in \{1, \ldots, i-1\}$ are subtracted by SIC.

In this system, the BS can successfully decode the received packets if the following three conditions hold: First, the packet with P_i can be successfully decoded if P_ℓ's for $\ell < i$ are all successfully decoded. Second, it is the only packet with P_i. If more than one packets with the same P_i are transmitted, all of them can not be decoded, which is called *power collision*. Along with the first condition, when a power collision occurs for the packets with P_i, the BS can not decode all the packets with P_ℓ for $\ell > i$. Third, regarding the packets with P_ℓ for $\ell > i$ (if they are transmitted) as interference, the BS decodes the packet with P_i if (3) holds.

For analytical simplicity, let us assume $N_0 = 1$ in (3). Let us further calculate each P_i for $i \in \{1, 2, \ldots, L\}$ when L packets are simultaneously transmitted with different TRP levels. If the equality holds in (3), we can iteratively obtain P_i from L to 1 as

$$P_i = \overline{\gamma}(1 + \overline{\gamma})^{L-i}, \quad \text{for } i \in \{1, 2, ..., L\}. \tag{4}$$

Then, it can be checked that the above P_i's are the minimum values when L packets are transmitted with different TRP levels.

B. Average Power Consumption and Access Opportunities

Let us denote by u_k the probability that the SNR of user i falls into the interval between τ_k and τ_{k-1}. It can be obtained as

$$u_k = \Pr[\tau_k \leq \gamma_i \leq \tau_{k-1}] = e^{-\tau_k} - e^{-\tau_{k-1}} \tag{5}$$

and $\tau_0 = \infty$.

For two levels of TRP, i.e., $L = 2$, let us denote p and q the (conditional) probability that user i chooses TRP P_1 or P_2 given that $\gamma_i \geq \tau_2$. Using (5), we obtain can these probabilities as

$$p = u_1/\phi \quad \text{and} \quad q = u_2/\phi. \tag{6}$$

If $u_1 = u_2$ in (6), using (5) we should have $e^{-\tau_2} - e^{-\tau_1} = e^{-\tau_1}$, which yields $\tau_1 = \tau_2 + \ln 2$. It is notable that the reason for assuming $u_1 = u_2$ is because the access opportunities with TRP levels P_1 and P_2 are made fair (or equal) with respect to SNR.

Let $\overline{P}_L$ denote the average transmit power consumption for L TRP level. Regardless of TRP levels, if the users have equal access opportunities, the average power consumption per (re)transmission for $L = 2$ can be expressed as

$$\begin{aligned} \overline{P}_2 &= \sum_{i=1}^{L} P_i \int_{\tau_i}^{\tau_{i-1}} \frac{1}{x} e^{-x} dx \cdot u_i \\ &= P_1 E_1(\tau_1) u_1 + P_2 (E_1(\tau_2) - E_1(\tau_1)) u_2 \\ &= \overline{\gamma} u_1 \left(\overline{\gamma} E_1(\tau_1) + E_1(\tau_2) \right), \end{aligned} \tag{7}$$

where $E_1(z) = \int_z^\infty \frac{e^{-t}}{t} dt$, we have used (4) and $u_1 = u_2$.

Let us consider the system with three levels of TRP: Similar to (6), with slight abuse of notation, for $L = 3$ we denote by p, q and y the (conditional) probabilities that user i chooses TRP P_1, P_2, or P_3, respectively, given that $\gamma_i \geq \tau_3$. We can get them as

$$p = u_1/\phi, \quad q = u_2/\phi, \quad \text{and} \quad y = u_3/\phi. \tag{8}$$

If $u_1 = u_2 = u_3$, i.e., equal access opportunities for TRP P_1, P_2, and P_3, we should have $e^{-\tau_2} - e^{-\tau_1} = e^{-\tau_1}$ and $e^{-\tau_3} - e^{-\tau_2} = e^{-\tau_2} - e^{-\tau_1}$. We get $\tau_2 = \tau_3 + \ln 1.5$ and $\tau_1 = \tau_3 + \ln 3$. The average power consumption per (re)transmission for $L = 3$ is also obtained as

$$\begin{aligned} \overline{P}_3 =& P_1 E_1(\tau_1) u_1 + P_2 (E_1(\tau_2) - E_1(\tau_1)) u_2 \\ &+ P_3 (E_1(\tau_3) - E_1(\tau_2)) u_3 \\ =& \overline{\gamma} u_1 \left((1 + \overline{\gamma}) \overline{\gamma} E_1(\tau_1) + \overline{\gamma} E(\tau_2) + E_1(\tau_3) \right). \end{aligned} \tag{9}$$

C. RA Procedure

Let us introduce the RA procedure of this NOMA system as follows: The BS broadcasts a (re)transmission probability r_t at downlink subslot of slot t. If a user has a packet to send, he is called backlogged user. When backlogged user i (re)transmits his packet, first he chooses one out of N channels randomly and gets his SNR γ_i of the channel. If $\gamma_i > \tau_L$, he adjusts his transmit power according to (1) and (re)transmits his packet with (re)transmission probability r_t based on Bernoulli trial. Accordingly, his packet (re)transmission probability is $r\phi$ if $r_t = r$. The BS decodes the received packets according to the decoding rules that we have introduced. We assume that the BS gives the channel outcome, i.e., (re)transmission success, to the backlogged users over the downlink right after the uplink transmission without an error. When the users cannot find their success, they will repeat this procedure at the next slot.

Let us consider an example. In (3), if three users (re)transmit their packet with TRP levels P_1, P_2, and P_3, the user's packet with TRP P_1 can be successfully received if

$$\frac{P_1}{P_2 + P_3 + N_0} \geq \overline{\gamma}, \tag{10}$$

in which two packets with P_2 and P_3 are treated as noise.

The packet with P_1 is subtracted from the entire received packet, and then the packet with TRP P_2 is decoded by treating P_3 as noise:

$$\frac{P_2}{P_3 + N_0} \geq \overline{\gamma} \quad \text{and} \quad \frac{P_3}{N_0} \geq \overline{\gamma}, \tag{11}$$

where user 2's is subtracted from the received packets when user 2's packet is decoded. Notice that in (10) and (11), we have $P_1 = \overline{\gamma}(1+\overline{\gamma})^2$, $P_2 = \overline{\gamma}(1+\overline{\gamma})$, and $P_3 = \overline{\gamma}$.

If SIC is imperfect, some residual power (interference) is left after TRP P_i is cancelled. Let ϵ_i be the maximum of residual power for TRP P_i which is assumed to be available at the BS who utilizes this information to determine TRP levels in the system initialization procedure. We then rewrite (11) as $\frac{P_2}{\epsilon_1+P_3+N_0}$ and $\frac{P_3}{\epsilon_1+\epsilon_2+N_0}$, respectively. Assuming $N_0 = 1$, we also rewrite (4) as $P_3 \geq \overline{\gamma}\big(1 + \sum_{i=1}^{2} \epsilon_i\big)$ and $P_2 \geq \overline{\gamma}\big(1 + \overline{\gamma} + \sum_{i=1}^{2} \epsilon_i + \epsilon_1\big)$. If ϵ_i is the maximum of residual power, we should set the TRP higher than (4) by taking into account these ϵ_i's. For imperfect SIC, while more transmit power is used, medium access control (MAC) layer performance might remain the same.

III. Analysis and Online Control Algorithm

A. Stability Analysis

First we show that if $r_t = r$ for $t = 0, 1, \ldots$, i.e., a fixed retransmission probability r used over time, the system with Poisson arrivals with mean rate λ (packets/slot) *always* becomes unstable no matter how many channels and TRP levels are used.

To begin with, let X_t be the number of backlogged users at the beginning of slot t, whereas S_t and A_t denote the number of users making a successful RA and that of users newly joining the backlog between slot t and $t+1$. In the course of time, X_t develops according to

$$X_{t+1} = X_t - S_t + A_t. \tag{12}$$

In (12), it can be seen that X_t is a Markov process.

Let us denote by π_m the (steady) state probability that the system has m backlogged users, i.e., $X_t = m$ as $t \to \infty$; that is, $\pi_m = \Pr[X_t = m]$. To find this, let us denote by R_t the number of backlogged users that (re)transmit their packet with a TRP to a channel at slot t. Let $p_{n,m}$ denote the state transition probability of X_t from slot t to the next slot, i.e., $p_{n,m} = \Pr[X_{t+1} = m | X_t = n]$. If a_n denotes the probability that n packets (or users) arrives to the system, i.e., n users become backlogged, we can write $p_{0,n}$ as

$$p_{0,n} = a_n. \tag{13}$$

For $n \geq 1$ and $0 \leq \ell \leq \overline{m}$, $p_{n,n-\ell}$ can be expressed as

$$p_{n,n-\ell} = \sum_{k=\ell}^{n} \sum_{i=\ell}^{\min(k,\overline{m})} \Pr[S_t = i | R_t = k] \Pr[R_t = k | X_t = n] \times \Pr[A_t = i - \ell], \tag{14}$$

where $\overline{m} = NL$ denotes the maximum number of packets that are successfully decoded at the same time. Furthermore, for $\ell \geq 1$ we can write $p_{,n+\ell}$ as

$$p_{n,n+\ell} = \sum_{k=0}^{n} \sum_{i=0}^{\min(k,\overline{m})} \Pr[S_t = i | R_t = k] \Pr[R_t = k | X_t = n] \times \Pr[A_t = i + \ell]. \tag{15}$$

Let $P = [p_{n,m}]$ and $\boldsymbol{\pi} = [\pi_m]$ denote the state transition probability matrix that takes $p_{n,m}$ as its element, and the (steady) state probability row vector with element π_m, respectively. Then, $\boldsymbol{\pi}$ can be obtained as

$$\boldsymbol{\pi} P = \boldsymbol{\pi} \quad \text{and} \quad \sum_{m=0}^{\infty} \pi_m = 1. \tag{16}$$

In order to show whether π_m exists or not, we need to show that Markov process X_t should be positive recurrent. To this end, we apply Foster-Lyapunov theorem [14], [15]: For an irreducible Markov process X_t with a state space $\mathbb{S}$, it is positive recurrent, i.e., *stable* in terms of returning to any state while starting from it within a finite time interval, if and only if there exists a function $L : \mathbb{S} \to R^+$ such that $L(i) \geq 0$ for all $i \in \mathbb{S}$, and for a finite set $\mathfrak{B} \subseteq \mathbb{S}$, the following two conditions are met:

1) For any $m \in \mathfrak{B}^c$, the drift ΔL_m is less than or equal to $-\epsilon$, where $\epsilon > 0$, i.e.,

$$\Delta L_m = \mathbb{E}[L(X_t) - L(X_{t-1}) | X_{t-1} = m] \leq -\epsilon, \tag{17}$$

2) For any $n \in \mathfrak{B}$, the drift is less than or equal to a constant $c < \infty$, i.e.,

$$\Delta L_m = \mathbb{E}[L(X_t) - L(X_{t-1}) | X_{t-1} = m] \leq c. \tag{18}$$

In applying Foster-Lyapunov theorem, suppose that $L(x) = x$ is used in (17) and $X_{t-1} = m \geq 1$. We can write (17) as

$$\begin{aligned}&\mathbb{E}[X_t - X_{t-1}|X_{t-1} = m] \\ &= \mathbb{E}[X_{t-1} - S_t + A_t - X_{t-1}|X_{t-1} = m] \\ &= \mathbb{E}[-S_t + A_t|X_{t-1} = m] \\ &= -\mathbb{E}[S_t|X_{t-1} = m] + \mathbb{E}[A_t|X_{t-1} = m] < 0.\end{aligned} \tag{19}$$

The term $D_m = -\mathbb{E}[S_t|X_{t-1} = m] + \mathbb{E}[A_t|X_{t-1} = m]$ is often called *drift*; it is notable that if $L(x) = x$ in (17), Foster-Lyapunov theorem corresponds to Pakes' lemma, known as stability lemma on p. 264 in [14]. The converse to Pakes' lemma is known as Kaplan's or instability lemma [16].

If A_t is independent of X_{t-1} for all t's owing to Poisson arrival assumption, we have

$$\mathbb{E}[A_t] < \mathbb{E}[S_t|X_{t-1} = m]. \tag{20}$$

It can be seen that (18) is satisfied if the packet arrivals are Poisson process with mean rate λ (packets/slot), i.e., $\mathbb{E}[A_t] = \lambda < \infty$. To find the the right-hand side (RHS) in (20), let us denote by $\Phi_k(m)$ the probability that k users (re)transmit their packet to one out of N channels at random if there are a total of m backlogged users. This also presumes that SNR of the channel chosen is greater than or equal to τ_L and k users pass Bernoulli trial with probability r. In addition, $\hat{S}_t$ denotes the number of packets successfully (re)transmitted to a channel at time t, whereas J_t denotes the number of packets transmitted to the channel. Then, the RHS of (20) can be expressed as

$$\begin{aligned}\mu_m^{(L)} &\triangleq \mathbb{E}[S_t|X_{t-1} = m] \\ &= N \sum_{k=0}^{m} \sum_{\ell=0}^{k} \ell \Pr[\hat{S}_t = \ell | J_t = k] \Phi_k(m),\end{aligned} \tag{21}$$

where $\Phi_k(m)$ can be expressed as

$$\begin{aligned}\Phi_k(m) = &\sum_{u=k}^{m} \sum_{n=k}^{u} \binom{n}{k} r^k (1-r)^{n-k} \\ &\times \binom{u}{n} \phi^n (1-\phi)^{u-n} \binom{m}{u} \left(\frac{1}{N}\right)^u \left(1 - \frac{1}{N}\right)^{m-u}.\end{aligned} \tag{22}$$

This can read that while u out of m users choose one of N channels at random. Due to i.i.d. assumption on γ_i in (2), the channel of n among u users is up (i.e. $\gamma_i > \tau_L$) with probability ϕ, whereas k out of n users makes a (re)transmission with probability r. Note that $\Phi_1(m)$ denotes the probability that only one user makes a (re)transmission to one of N channel at random.

To write (21) in a concise form as m grows, we need the following lemma.

Lemma 1: As m becomes large, $\Phi_k(m)$ in (22) can be approximated as a Poisson distribution with mean $r\phi m/N$ (packets/slot), i.e.,

$$\Phi_k(m) \approx \Lambda_k(m) \triangleq \frac{\left(\frac{r\phi m}{N}\right)^k}{k!} e^{-\frac{r\phi m}{N}}. \tag{23}$$

Proof: For $k = 0$, let us look at $\Phi_0(m)$:

$$\begin{aligned}\Phi_0(m) &= \sum_{u=0}^{m} \sum_{n=0}^{u} (1-r)^n \binom{u}{n} \phi^n (1-\phi)^{u-n} \\ &\quad \times \binom{m}{u} \left(\frac{1}{N}\right)^u \left(1 - \frac{1}{N}\right)^{m-u} \\ &= \sum_{u=0}^{m} (1 - r\phi)^u \binom{m}{u} \left(\frac{1}{N}\right)^u \left(1 - \frac{1}{N}\right)^{m-u} \\ &= \left(1 - \frac{r\phi}{N}\right)^m \approx e^{-\frac{r\phi m}{N}},\end{aligned} \tag{24}$$

where the binomial theorem is used, i.e., $\sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i} = (a+b)^n$ and we also used $(1-x)^n \approx e^{-xn}$ for some small x. It can read that u backlogged users choose a channel with probability $\frac{1}{N}$. Among them, n users' channel is up, but none of them (re)transmit.

We can also find $\Phi_1(m)$ in (25) given at the top of the next page, where we assume that m becomes large, as well. Similarly, $\Phi_2(m)$ can be expressed as

$$\begin{aligned}\Phi_2(m) = &\frac{m(m-1)}{2} \left(\frac{\phi r}{N}\right)^2 \left(1 - \frac{r\phi}{N}\right)^{m-2} \\ \approx &\frac{1}{2} \left(\frac{r\phi m}{N}\right)^2 e^{-\frac{r\phi m}{N}},\end{aligned} \tag{26}$$

where we have assumed that $m(m-1) \approx m^2$ as m becomes large. This completes the proof. ∎

Let $\Delta_k^{(L)}$ denote the expected number of packets successfully decoded in a slot when k packets are (re)transmitted at the same slot in the system with L levels of TRP.

$$\Delta_k^{(L)} = \sum_{\ell=0}^{k} \ell \Pr[\hat{S}_t = \ell | J_t = k]. \tag{27}$$

Then, $\mu_m^{(L)}$ can be expressed as

$$\begin{aligned}\mu_m^{(L)} &= N \sum_{k=1}^{m} \Delta_k^{(L)} \Phi_k(m) \approx N \sum_{k=1}^{m} \Delta_k^{(L)} \Lambda_k(m) \\ &= N e^{-\frac{r\phi m}{N}} \sum_{k=1}^{m} \Delta_k^{(L)} \frac{\left(\frac{r\phi m}{N}\right)^k}{k!},\end{aligned} \tag{28}$$

where we have used Lemma 1.

Let us now find $\Delta_k^{(L)}$. For $L = 2$, we assume that a user chooses P_1 (or P_2) with probability p (or q). Then, we can get $\Delta_1^{(2)} = 1$ and $\Delta_2^{(2)}$ as

$$\Delta_2^{(2)} = 2 \binom{2}{1} pq. \tag{29}$$

It is not difficult to see $\Delta_k^{(2)} = 0$ for $k > 2$ due to (3).

If $L = 3$, let us recall p, q, and y in (8), i.e., the probability that a user chooses P_1, P_2 and P_3, respectively. One can get $\Delta_1^{(3)} = 1$. When two users (re)transmit, $\Delta_2^{(3)}$ is obtained as

$$\Delta_2^{(3)} = 2 \left[\binom{2}{1} pq + \binom{2}{1} py + \binom{2}{1} qy \right]. \tag{30}$$

$$\begin{aligned}
\Phi_1(m) &= \sum_{u=1}^{m}\sum_{n=1}^{u}\binom{n}{1} r\,(1-r)^{n-1}\binom{u}{n}\phi^n(1-\phi)^{u-n}\binom{m}{u}\left(\frac{1}{N}\right)^u\left(1-\frac{1}{N}\right)^{m-u}\\
&= \sum_{u=1}^{m}\sum_{n=1}^{u}\frac{nr}{1-r}\binom{u}{n}\left((1-r)\phi\right)^n(1-\phi)^{u-n}\binom{m}{u}\left(\frac{1}{N}\right)^u\left(1-\frac{1}{N}\right)^{m-u}\\
&= \sum_{u=1}^{m}\sum_{n=1}^{u}\frac{r\phi\cdot u\cdot(u-1)!}{(n-1)!(u-1-(n-1))!}\left((1-r)\phi\right)^{n-1}(1-\phi)^{u-1-(n-1)}\binom{m}{u}\left(\frac{1}{N}\right)^u\left(1-\frac{1}{N}\right)^{m-u}\\
&= \sum_{u=1}^{m} r\phi\,(1-r\phi)^{u-1}\,u\binom{m}{u}\left(\frac{1}{N}\right)^u\left(1-\frac{1}{N}\right)^{m-u} = \frac{r\phi m}{N}\left(1-\frac{r\phi}{N}\right)^{m-1} \approx \frac{r\phi m}{N}e^{-\frac{r\phi m}{N}}. \qquad (25)
\end{aligned}$$

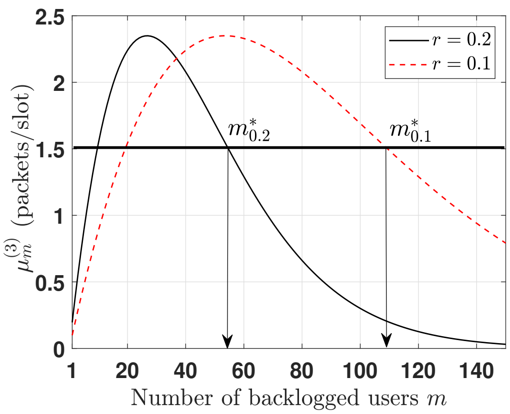

Fig. 1. Mean drift of the system with $L = 3$.

In case of three (re)transmitting users, $\Delta_3^{(3)}$ can be expressed as

$$\Delta_3^{(3)} = \left[3\frac{3!}{1!1!1!}pqy + \frac{3!}{1!0!2!}pq^0y^2\right], \qquad (31)$$

where the first term explains the successful (re)transmission of three users, each of whom chooses different TRP levels. The second term shows one successful transmission of one user with TRP P_1 while two users (re)transmit with TRP P_3.

Especially for $L = 3$, one user with TRP P_1 can make a successful (re)transmission, even if three users (re)transmit at the same slot with P_3; that is, $\frac{\overline{\gamma}(1+\overline{\gamma})^2}{3\overline{\gamma}+N_0} \geq \overline{\gamma}$ with $N_0 = 1$ and $\overline{\gamma} \geq 1$ in (3). We then get $\Delta_4^{(3)}$ as

$$\Delta_4^{(3)} = \frac{4!}{1!0!3!}py^3. \qquad (32)$$

In addition, we have $\Delta_k^{(3)} = 0$ for $k > 4$. Notice that the number of terms in $\Delta_k^{(L)}$ is *at least* $\binom{L}{k}$. As L is raised, more terms get involved as long as (3) holds such that the analysis becomes intractable. We shall see later that such an analytical complexity arises when an optimal (re)transmission probability is derived for $L > 2$.

Now, let us discuss the stability or positive recurrence of X_t. In Fig. 1, we depict (28) for $r = 0.1$ and 0.2, respectively when $\phi = 1$ and $L = 3$. The horizontal straight line represents the mean of packet arrival rate $\mathbb{E}[A_t] = \lambda$. If we set $\mathfrak{B}^c = \{m | m \geq m^*_{0.2}$ (or $m^*_{0.1}$)$\}$, it can be observed that $\lambda > \mu_m^{(3)}$ so

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Algorithm 1 NOMA Real-time transmission algorithm

Initialize $\widetilde{X}_0 = 10$, $\widetilde{\lambda}_0 = 1$, and $\alpha = 0.99$. Repeat the following steps at the beginning of each RA slot.

1: $\widetilde{\lambda}_t = \alpha\widetilde{\lambda}_{t-1} + (1-\alpha)S_t$
2: **if** $L = 2$ **then**
3: $\quad\widetilde{X}_t = \widetilde{X}_{t-1} + 0.4543N - 1.8685\mathbb{I} - S_t + \widetilde{\lambda}_t$
4: $\quad$Broadcast $r_t = \min\left(1, \frac{\sqrt{2}N}{\widetilde{X}_t\phi}\right)$
5: **else if** $L = 3$ **then**
6: $\quad\widetilde{X}_t = \widetilde{X}_{t-1} + 0.36N - 2.1442\mathbb{I} - S_t + \widetilde{\lambda}_t$
7: $\quad$Broadcast $r_t = \min\left(1, \frac{1.7841N}{\widetilde{X}_t\phi}\right)$
8: **end if**

---

that (17) or (20) can be violated for $m > m^*_{0.2}$. As long as a fixed retransmission probability r is used, this always happens since $\mu_m^{(L)} \to 0$ as $m \to \infty$. As shown in (14) and (15), a state $m > m^*_{0.2}$ in Markov chain for X_t is reachable from any state belonging to set $\mathfrak{B}$. Therefore, the system becomes unstable eventually however small λ is. The next subsection shows how to stabilize the system by controlling r_t over time.

B. Online Control Algorithm

The objective of NOMA based real-time transmission in Algorithm 1 is to stabilize the system over time, while maximizing the throughput. First, we introduce how Algorithm 1 works. In Algorithm 1, $\widetilde{\lambda}_t$ and $\widetilde{X}_t$ denote the (estimated) average of new packet arrivals and of backlogged users at slot t. In line 1, the BS updates $\widetilde{\lambda}_t$ based on an autoregressive (AR) model by assuming that the average packet arrivals to the system would be equal to the average number of packets served by the system in the past. Although such $\widetilde{\lambda}_t$ may not accurately predict future arrivals, especially when sudden change in the packet arrivals happens to the system, this information is still helpful in estimating $\widetilde{X}_t$.

Let $\mathbb{I}$ denote the number of idle channels (observed); that is, the number of channels, where no packet is (re)transmitted at all at slot t. After observing $\mathbb{I}$, the BS updates $\widetilde{X}_t$ as in lines 3 and 6 for $L = 2$ and $L = 3$, respectively. Lines 3 and 6 indicate that the larger the $\mathbb{I}$, the smaller number of backlogged users in the system. Finally, it broadcasts retransmission probability r_t at the downlink subslot of slot t. Then, backlogged users (re)transmit their packet with probability r_t.

Let us derive the update equations in lines 3 and 6 in Algorithm 1. First, we find r to maximize $\mu_m^{(L)}$ in (28). This is used in lines 4 and 7 in Algorithm 1.

Proposition 1: When the maximizer of $\mu_m^{(L)}$ is denoted by r_m^*, it can be obtained as

$$r_m^* = \min\left(1, \frac{\theta N}{m\phi}\right), \tag{33}$$

where ϕ is given in (2), and θ depends on L. It is found as

$$\theta = \begin{cases} \sqrt{2}, & L=2, \\ 1.7841, & L=3. \end{cases} \tag{34}$$

Proof: We make use of $\frac{d\mu_m^{(L)}}{dr} = 0$ to find r_m^*. For $L=2$, one gets

$$\frac{d\mu_m^{(2)}}{dr} = \phi m e^{-\frac{r\phi m}{N}} \left(\Delta_1^{(2)}\left(1-\frac{r\phi m}{N}\right) + \Delta_2^{(2)}\left(1-\frac{r\phi m}{2N}\right)\frac{r\phi m}{N}\right). \tag{35}$$

If $p=q=\frac{1}{2}$, (35) becomes

$$\frac{d\mu_m^{(2)}}{dr} = \phi m e^{-\frac{r\phi m}{N}} \left(1-\frac{1}{2}\left(\frac{\phi m}{N}\right)^2 r^2\right). \tag{36}$$

From $\frac{d\mu_m^{(2)}}{dr} = 0$ for $r \in (0,1)$, we can see that $r = \sqrt{2}N/(m\phi)$ is the maximizer of $\mu_m^{(2)}$.

Let us find r_m^* for $\mu_m^{(3)}$ for $L=3$. The derivative of (28) with respect to r becomes

$$\begin{aligned}\frac{d\mu_m^{(3)}}{dr} =& \phi m e^{-\frac{r\phi m}{N}}\Bigg[\Delta_1^{(3)}\left(1-\frac{r\phi m}{N}\right) \\ &+ \Delta_2^{(3)}\left(1-\frac{1}{2}\frac{r\phi m}{N}\right)\left(\frac{r\phi m}{N}\right) \\ &+ \frac{\Delta_3^{(3)}}{2}\left(1-\frac{1}{3}\frac{r\phi m}{N}\right)\left(\frac{r\phi m}{N}\right)^2 \\ &+ \frac{\Delta_4^{(3)}}{6}\left(1-\frac{1}{4}\frac{r\phi m}{N}\right)\left(\frac{r\phi m}{N}\right)^3\Bigg].\end{aligned} \tag{37}$$

If $p=q=y=\frac{1}{3}$, (37) becomes

$$\frac{d\mu_m^{(3)}}{dr} = \phi m e^{-\frac{r\phi m}{N}} f(r), \tag{38}$$

where $f(r)$ is expressed as

$$\begin{aligned}f(r) =& -\frac{1}{486}\left(\frac{\phi m}{N}\right)^4 r^4 - \frac{59}{486}\left(\frac{\phi m}{N}\right)^3 r^3 \\ &- \frac{5}{18}\left(\frac{\phi m}{N}\right)^2 r^2 + \frac{1}{3}\frac{\phi m}{N} r + 1.\end{aligned} \tag{39}$$

To find the roots of $\frac{d\mu_m^{(3)}}{dr} = 0$, we need to solve $f(r)=0$. We assume a factored form of $f(r)=0$ as

$$(ar+1)(br^3+cr^2+dr+1) = 0. \tag{40}$$

Its expanded form can be expressed as

$$abr^4 + (b+ac)r^2 + (ad+c)r^2 + (a+d)r + 1 = 0. \tag{41}$$

Then, a, b, c, and d in (41) are related to the coefficients of (39) as

$$\begin{aligned} &ab = -\frac{1}{486}\left(\frac{\phi m}{N}\right)^4, \; ac+b = -\frac{59}{486}\left(\frac{\phi m}{N}\right)^3, \\ &ad+c = -\frac{5}{18}\left(\frac{\phi m}{N}\right)^2, \quad a+d = \frac{1}{3}\frac{\phi m}{N}.\end{aligned} \tag{42}$$

According to (42), we assume that

$$a = x_1\frac{\phi m}{N}, \qquad b = x_2\left(\frac{\phi m}{N}\right)^3, \tag{43}$$

$$c = x_3\left(\frac{\phi m}{N}\right)^2, \text{ and } d = x_4\frac{\phi m}{N}. \tag{44}$$

Using (43) and (44), we can conceal the terms $(\frac{m}{L})^i$ for $i = 1, 2, 3$, and 4 in (42) so that (42) can be rewritten in terms of x_1, x_2, x_3 and x_4 as

$$x_1x_2 = -\frac{1}{486}, \quad x_2 + x_1x_3 = -\frac{59}{486} \tag{45}$$

$$x_1x_4 + x_3 = -\frac{5}{18}, \qquad x_1 + x_4 = \frac{1}{3}. \tag{46}$$

Combining two equations in (45) with respect to x_2, we have

$$x_1^2x_3 + \frac{59}{486}x_1 = \frac{1}{486}. \tag{47}$$

Again combining two equations in (46) with respect to x_4, we have

$$-x_1^2 + \frac{1}{3}x_1 + x_3 = -\frac{5}{18}. \tag{48}$$

Using (48) we can write (47) as a function of x_1:

$$x_1^4 - \frac{1}{3}x_1^3 - \frac{5}{18}x_1^2 + \frac{59}{486}x_1 - \frac{1}{486} = 0. \tag{49}$$

By getting rid of symbols, ϕ, m and N in (39), we have (49). Now, (49) can be solved numerically: The roots of (49) are -0.5605, 0.0177, $0.4381 \pm 0.1256j$ for $j = \sqrt{-1}$. One real root, i.e., $x_1 = -0.5605$ gives a positive r from $ar+1=0$ in (40); that is, $r = N/(0.5605m\phi) = 1.7841N/(m\phi)$. ■

In (33), the constant $\theta = 1.7841$ for $L=3$ becomes larger than $\theta = \sqrt{2}$ for $L=2$. It means that more users are encouraged to (re)transmit as the number of TRP levels increases. If the threshold for (re)transmission τ_L in (2) is raised, ϕ becomes small. In other words, a higher τ_L suppresses access opportunity, while saving transmit power. Such a suppressed access opportunity is compensated in (33), since r_m^* is increased due to smaller ϕ.

If we can realize $r_t = r_m^*$ over time for $X_t = m$, the following proposition shows that Algorithm 1 can stabilize the system under a certain condition.

Proposition 2: The system is stabilized, i.e., X_t is positive recurrent, if the mean packet arrival rate satisfies the following inequality:

$$\lambda < \mu_m^* = \zeta N, \tag{50}$$

where ζ can be expressed as

$$\zeta = \begin{cases} 0.5896, & L=2, \\ 0.7832, & L=3. \end{cases} \tag{51}$$

Proof: Substituting (33) into (28), we can get μ_m^* for all m's as

$$\mu_m^* = \mathbb{E}[S_t|X_t = m] = \begin{cases} 0.5896N, & L = 2, \\ 0.7832N, & L = 3. \end{cases} \tag{52}$$

Thus, if $\lambda < \mu_m^*$, (20) holds, which completes the proof. ■

Let us consider the update equations given in lines 3 and 6, respectively. To realize Proposition 1, the BS should know $X_t = m$ every slot t and broadcast r_m^* accordingly. However, it is not possible for the BS to know the exact value of X_t every slot t. Therefore, the BS estimates $\mathbb{E}[X_t]$ instead of X_t. To this end, we make use of the algorithm proposed in [13]. Let us remind that $\mathbb{I}$ is the number of channels not used at slot t. Let us denote $\mathbb{E}[X_t]$ for $t = 1, 2, \ldots$, by β. The underlying concept of the algorithm in [13] is to estimate $\mathbb{E}[X_t]$ based on $\mathbb{I}$. Let us denote by $P_R(k, \ell) = \Pr[\mathbb{I} = k, X_t = \ell]$ the probability that the system has ℓ backlogged users at a slot, say slot t, when k channels are idle. We use the same notation given (12) in [13] as a reference. The main difference between (12) in [13] and ours is to take into account the probability ϕ that a backlogged user's channel is up. The conditional expectation of X_t given $\mathbb{I} = k$ is then expressed as

$$\mathbb{E}[X_t|\mathbb{I} = k] = \frac{\sum_{\ell=0}^{\infty} \ell P_R(k, \ell)}{\sum_{\ell=0}^{\infty} P_R(k, \ell)}. \tag{53}$$

The numerator and denominator of (53) are obtained in (54) and (55), respectively, which are given on the top of the next page. It is important to note that the number of backlogged users is assumed to follow a Poisson distribution with mean β (users/slot) in (54) and (55):

$$\Pr[X_{t-1} = \ell] = \frac{\beta^\ell}{\ell!} e^{-\beta}, \tag{56}$$

which plays a role of the *a priori* probability distribution, i.e., *belief* on the state.

Furthermore, $P_I(k|m)$ in (54) and (55) denotes the probability that k channels are not used at all among a total of N channels, when m backlogged users (re)transmit. In [13] it is given by

$$P_I(k|m) = \sum_{i=0}^{N-k} \binom{N}{k} (-1)^i \binom{N-k}{i} \left(1 - \frac{k+i}{N}\right)^m. \tag{57}$$

Now, it can read in (54) that when n out of ℓ backlogged users' SNR is good enough to (re)transmit, only m out of them (re)transmit, but k channels are not used.

Substituting (54) and (55) into (53), we get the conditional expectation of β given an observation $\mathbb{I} = k$ as

$$\mathbb{E}[X_t|\mathbb{I} = k] = \beta \left[1 - \phi r + \phi r \left(1 - \frac{k}{N}\right)\left(1 - e^{-\frac{\phi r \beta}{N}}\right)^{-1}\right]. \tag{58}$$

Starting from the mean β of the *a priori* distribution in (56), it is updated by (58) based on the observation on $\mathbb{I}$. Indeed, it is the mean of the *a posteriori* distribution.

If r takes a form of $\theta N/\beta$ as in (33) and is substituted into (58), we can write (58) as

$$\mathbb{E}[X_t|\mathbb{I} = k] = \beta + \frac{\theta e^{-\theta} N - \theta k}{1 - e^{-\theta}}. \tag{59}$$

When we use $\theta = \sqrt{2}$ for $L = 2$ and 1.7841 for $L = 3$ in (34), (59) can be expressed as

$$\mathbb{E}[X_t|\mathbb{I} = k] = \begin{cases} \beta + (0.4543N - 1.8685k), & L = 2, \\ \beta + (0.36N - 2.1442k), & L = 3. \end{cases} \tag{60}$$

When β and k in (60) are replaced with $\widetilde{X}_t$ and $\mathbb{I}$, (60) can be found in lines 3 and 6, respectively.

C. Delay Analysis

Let us examine the (approximate) average delay of the users with the proposed algorithm when the system has Poisson arrivals with mean rate λ (packets/slot). To do this, instead of capturing the Markov process X_t in (12) with a discrete-time Markov chain, we approximate it as a continuous-time Markov chain, i.e., a simple M/M/1 queueing system. Without loss of generality, when the packet arrival process follows a Poisson process with mean rate λ_m at state m, the flow balance equation for π_m can be written as

$$\lambda_m \pi_m = \mu_{m+1}^{(L)} \pi_{m+1} \Rightarrow \pi_{m+1} = \frac{\lambda_m}{\mu_{m+1}^{(L)}} \pi_m, \tag{61}$$

where the mean input rate to the system and the mean output rate are considered. If the packet arrival process follows a Poisson process with mean rate λ regardless of m, we have $\lambda_m = \lambda$. For $n \in \mathbb{Z}^+$, in terms of π_0, we can rearrange (61) as

$$\pi_n = \pi_0 \prod_{m=0}^{n-1} \frac{\lambda_m}{\mu_{m+1}^{(L)}}. \tag{62}$$

Using $\sum_{n=0}^{\infty} \pi_n = 1$, we get π_0 as

$$\pi_0 = \left(1 + \sum_{n=1}^{\infty} \prod_{m=0}^{n-1} \frac{\lambda_m}{\mu_{m+1}^{(L)}}\right)^{-1}. \tag{63}$$

Then, the system throughput can be obtained as

$$\overline{\tau} = \sum_{m=0}^{\infty} \mu_m^{(L)} \pi_m, \tag{64}$$

where $\mu_0^{(L)} = 0$. Based on (50), it can be seen that $\mu_m^{(L)} = \mu_m^* = \zeta N$ regardless of m. Therefore, we have $\overline{\tau} = \zeta N$.

Proposition 3: In the system with Poisson traffic with mean rate λ (users/slot), the average RA delay with Algorithm 1 is expressed as

$$\overline{d} = \frac{1}{\zeta N - \lambda} + 0.5, \tag{65}$$

where 0.5 is added to account for the slot synchronization delay of Poisson arrivals and ζ is given in (51).

$$
\begin{aligned}
P_R(k) = \sum_{\ell=0}^{\infty} P_R(k,\ell) &= \sum_{\ell=0}^{\infty}\sum_{n=0}^{\ell}\sum_{m=0}^{n} P_I(k|m)\binom{n}{m} r^m (1-r)^{n-m}\binom{\ell}{n}\phi^n(1-\phi)^{\ell-n}\Pr[X_{t-1}=\ell] \\
&= \sum_{\ell=0}^{\infty}\sum_{n=0}^{\ell}\sum_{m=0}^{n}\sum_{i=0}^{N-k}\left[\binom{N}{k}(-1)^i\binom{N-k}{i}\left(1-\frac{k+i}{N}\right)^m\right]\binom{n}{m} r^m(1-r)^{n-m}\binom{\ell}{n}\phi^n(1-\phi)^{\ell-n}\frac{\beta^\ell}{\ell!}e^{-\beta} \\
&= \sum_{\ell=0}^{\infty}\sum_{i=0}^{N-k}\sum_{n=0}^{\ell}\binom{N}{k}(-1)^i\binom{N-k}{i}\sum_{m=0}^{n}\binom{n}{m}\left[\left(1-\frac{k+i}{N}\right)r\right]^m(1-r)^{n-m}\binom{\ell}{n}\phi^n(1-\phi)^{\ell-n}\frac{\beta^\ell}{\ell!}e^{-\beta} \\
&= \sum_{\ell=0}^{\infty}\sum_{i=0}^{N-k}\binom{N}{k}(-1)^i\binom{N-k}{i}\sum_{n=0}^{\ell}\left[1-\frac{(k+i)r}{N}\right]^n\binom{\ell}{n}\phi^n(1-\phi)^{\ell-n}\frac{\beta^\ell}{\ell!}e^{-\beta} \\
&= \sum_{\ell=0}^{\infty}\sum_{i=0}^{N-k}\binom{N}{k}(-1)^i\binom{N-k}{i}\left[1-\frac{(k+i)r\phi}{N}\right]^\ell\frac{\beta^\ell}{\ell!}e^{-\beta} \\
&= \sum_{i=0}^{N-k}\binom{N}{k}(-1)^i\binom{N-k}{i}e^{-\frac{(k+i)\phi r\beta}{N}} = \binom{N}{k}e^{-\frac{k\phi r\beta}{N}}\sum_{i=0}^{N-k}\binom{N-k}{i}\left(-e^{-\frac{\phi r\beta}{N}}\right)^i \\
&= \binom{N}{k}\left(e^{-\frac{\phi r\beta}{N}}\right)^k\left(1-e^{-\frac{\phi r\beta}{N}}\right)^{N-k}.
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
\sum_{\ell=0}^{\infty}\ell P_R(k,\ell) &= \sum_{\ell=0}^{\infty}\ell\sum_{n=0}^{\ell}\sum_{m=0}^{n}\sum_{i=0}^{N-k}\left[\binom{N}{k}(-1)^i\binom{N-k}{i}\left(1-\frac{k+i}{N}\right)^m\right]\binom{n}{m}r^m(1-r)^{n-m}\binom{\ell}{n}\phi^n(1-\phi)^{\ell-n}\frac{\beta^\ell}{\ell!}e^{-\beta} \\
&= \sum_{\ell=0}^{\infty}\ell\sum_{i=0}^{N-k}\sum_{n=0}^{\ell}\binom{N}{k}(-1)^i\binom{N-k}{i}\sum_{m=0}^{n}\binom{n}{m}\left[\left(1-\frac{k+i}{N}\right)r\right]^m(1-r)^{n-m}\binom{\ell}{n}\phi^n(1-\phi)^{\ell-n}\frac{\beta^\ell}{\ell!}e^{-\beta} \\
&= \sum_{\ell=0}^{\infty}\ell\sum_{i=0}^{N-k}\binom{N}{k}(-1)^i\binom{N-k}{i}\sum_{n=0}^{\ell}\left[1-\frac{(k+i)r}{N}\right]^n\binom{\ell}{n}\phi^n(1-\phi)^{\ell-n}\frac{\beta^\ell}{\ell!}e^{-\beta} \\
&= \sum_{i=0}^{N-k}\binom{N}{k}(-1)^i\binom{N-k}{i}\left(\beta-\frac{(k+i)\phi r\beta}{N}\right)\sum_{\ell=0}^{\infty}\frac{\left[\left(1-\frac{(k+i)\phi r}{N}\right)\beta\right]^{\ell-1}}{(\ell-1)!}e^{-\beta} \\
&= \binom{N}{k}e^{-\frac{k\phi r\beta}{N}}\sum_{i=0}^{N-k}\binom{N-k}{i}\left(-e^{-\frac{\phi r\beta}{N}}\right)^i\left[\frac{(N-k\phi r)\beta}{N}-\frac{i\phi r\beta}{N}\right] \\
&= \binom{N}{k}e^{-\frac{k\phi r\beta}{N}}\beta\left[\frac{(N-k\phi r)}{N}\sum_{i=0}^{N-k}\binom{N-k}{i}\left(-e^{-\frac{\phi r\beta}{N}}\right)^i-\sum_{i=0}^{N-k}\binom{N-k}{i}\left(-e^{-\frac{\phi r\beta}{N}}\right)^i\frac{i\phi r}{N}\right] \\
&= \binom{N}{k}e^{-\frac{k\phi r\beta}{N}}\left(1-e^{-\frac{\phi r\beta}{N}}\right)^{N-k}\beta\left[\frac{(N-k\phi r)}{N}+\frac{\phi r}{N}(N-k)\left(1-e^{-\frac{\phi r\beta}{N}}\right)^{-1}e^{-\frac{\phi r\beta}{N}}\right] \\
&= \binom{N}{k}e^{-\frac{k\phi r\beta}{N}}\left(1-e^{-\frac{\phi r\beta}{N}}\right)^{N-k}\beta\left[1-\phi r+\phi r\left(1-\frac{k}{N}\right)\left(1-e^{-\frac{\phi r\beta}{N}}\right)^{-1}\right].
\end{aligned} \tag{55}
$$

Proof: Let us recall that when r_m^* in (33) is used, we have $\mu_m = \mu_m^*$ given in (50). When $\lambda_m = \lambda$, we can write the system utilization ρ as

$$\rho = \frac{\lambda_m}{\mu_{m+1}} = \frac{\lambda}{\zeta L}. \tag{66}$$

For a stable system we should have $\rho < 1$, which means $\lambda < \zeta L$. Under this condition, from (62) we get π_n as

$$\pi_n = (1-\rho)\rho^n. \tag{67}$$

Let $\overline{n}$ and $\overline{d}$ denote the average number of backlogged UEs and the mean access delay. Using (62), (63), and (66), we can find $\overline{n}$ as

$$\overline{n} = \frac{\rho}{1-\rho} = \frac{\lambda}{\zeta L-\lambda}. \tag{68}$$

Using Little's result, i.e., $\lambda\overline{d} = \overline{n}$, we get (65). ■

It is important to note that the stability condition in (50), the update equation in (60), and the average RA delay in (65) are invariant of ϕ. It depends on only N and L.

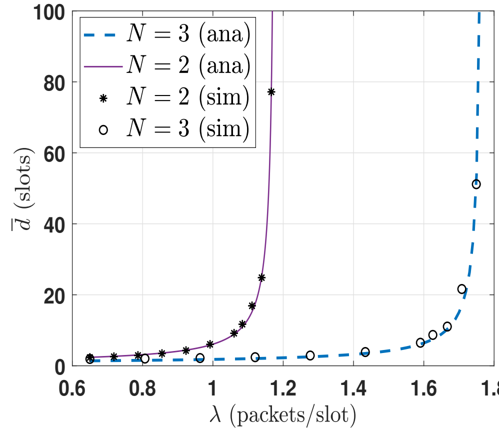

Fig. 2. Average RA delay of the system with $L = 2$.

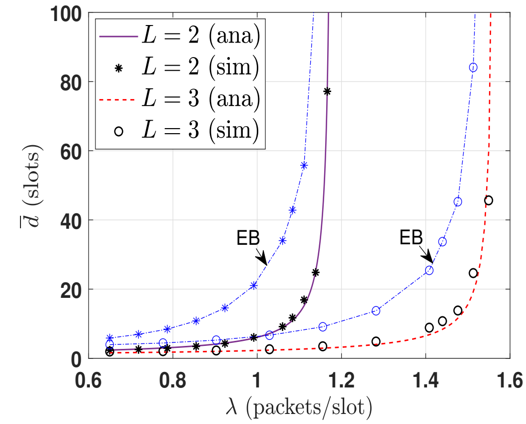

Fig. 4. Average RA delay of the system with $N = 2$.

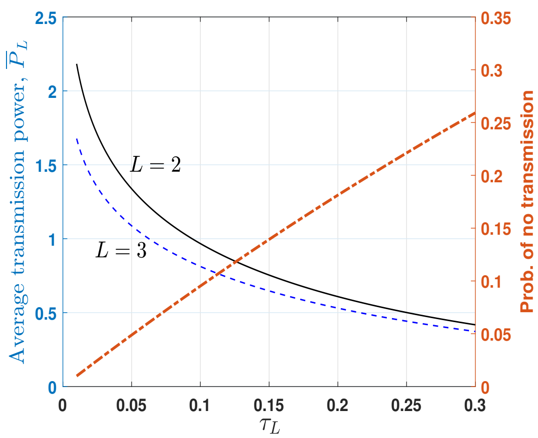

Fig. 3. Average power consumption per (re)transmission.

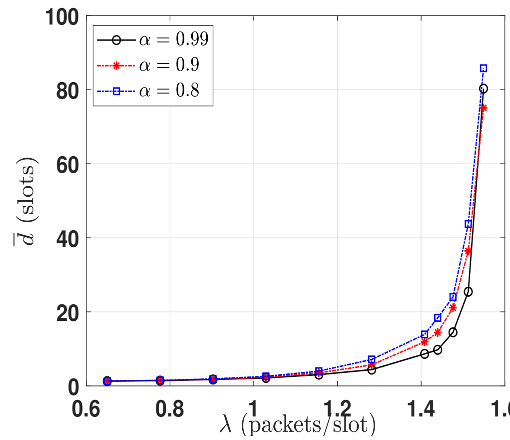

Fig. 5. The effect of α on the average RA delay with $N = 2$ and $L = 3$.

IV. Numerical Results

A simulation program is made with Matlab. The simulation run length is set to 10^6 (slots) and we obtain the time-averaged results. Let us recall that the channel fading dependant received SNR variation is considered in each transmission slot by taking the channel gain $h_i \sim \mathcal{CN}(0, 1)$ for each user.

Fig. 2 presents the average RA delay of the system with $\phi = 0.6065$ and two TRP levels, i.e., $L = 2$ by increasing the number of independent channels $N = 2$ to 3. The solid and dashed lines depict the analytical results for $N = 2$ and 3 in (65), respectively, whereas symbols show the simulation results obtained with the proposed Algorithm. Good agreements between the analytical and simulation results can be observed. As proven in Proposition 2, it can be observed that the system admits 0.5896 times as large the mean packet arrival rate by employing each additional channel. It is also stable as long as $\lambda < \mu_m^*$.

In Fig. 3 we compare the average power consumptions $\overline{P}_L$ for $L = 2$ and 3 for $\overline{\gamma} = 1$, whereas the probability of no transmission ϕ in (2) due to $\gamma_i < \tau_L$ is depicted as well. On the horizontal axis, we apply the same threshold for no transmission in (1) either $L = 2$ or 3. Thus, the probability of no transmission remains the same for two and three TRP levels. It increases as the threshold gets higher, which lowers the average transmission power. On the other hand, as far as small-scale fading is concerned in (1), an increase in the number of TRP levels reduces $\overline{P}_L$. In other words, coarse granularity in TRP levels, say $L = 2$, can increase the average power consumption compared to $L = 3$.

Fig.4 depicts the average RA delay as L increases. It can be seen that RA delay is significantly improved by taking one more TRP level, whereas $\overline{P}_L$ is reduced in Fig. 3. Together with the result in Fig. 4, we come to the conclusion that the system with more TRP levels provides a shorter average delay in a power-efficient way. In addition, the proposed algorithm is compared with an exponential backoff (EB) algorithm that is denoted by EB in Fig. 4. It works as follows: When a user has a packet to send, he picks up a random integer in an interval between 1 and $W_{\min}$ and counts it down by one every slot; $W_{\min}$ is the minimum window size. If the counter hits zero, he chooses one out of N channels at random and sees whether the channel is up. If so, he transmits the packet. Otherwise, he keeps doing this until transmitting the packet; that is, at the next slot, he selects one channel at random and looks for whether it is up. If his transmission fails, he chooses an integer randomly between 1 and $\min(W^R, W_{\max})$,

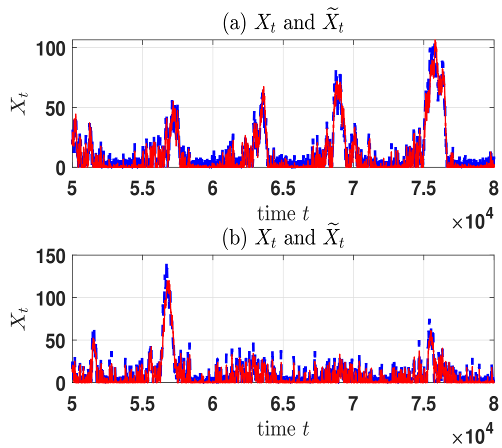

Fig. 6. Backlog tracking in the system with $L = 3$ and $N = 2$.

where R is the number of retransmissions, W is an increasing factor, and $W_{\max}$ is the maximum window size, respectively. He adds it to $W_{\min}$ and counts it down every slot for the next retransmission. If this counter becomes zero again, he picks up a channel at random and see whether it is up as before. In this EB algorithm, the window size increases exponentially every retransmission. We set $W_{\min} = 4$, $W = 2$, and $W_{\max} = 1024$, i.e., binary EB (BEB) algorithm. It can be seen that the RA delay of BEB algorithm is much larger than that of the proposed algorithm, while the BS does not need to control. It may be expected that W, $W_{\min}$, and $W_{\max}$ might be optimized given L, N, and λ to reduce the RA delay, which is beyond our scope.

In Fig. 5, the effect of smoothing parameter α in line 1 of Algorithm 1 is examined. We set $N = 2$, and $L = 3$. It is notable that as α gets closer to 1, the system assumes that λ_t vary slowly. It can be seen that the result with $\alpha = 0.99$ is better than others, which is the reason that we choose $\alpha = 0.99$.

In practice, the traffic (particularly its mean) may change hourly and daily in a periodic pattern [17]. To see whether the proposed algorithm can adapt to such a time-varying traffic, Fig. 6 presents how the proposed algorithm can keep track of the actual number of backlogged users X_t in terms of $\widetilde{X}_t$. We consider a Poisson process with time-varying mean λ_t (packets/slot) in the simulation: $\lambda_t = 0.3\cos(0.001t) + 1.25$ in Fig. 6(a), which might be able to emulate the traffic presented in [17]. The mean input rate of this time-varying Poisson process varies from 0.95 (packets/slot) to 1.55. We add an additional fast-varying component in Fig. 6(b); that is, $\lambda_t = 0.2\cos(0.001t) + 0.3\cos(0.05t) + 1.25$. Note that these two traffic models may capture some periodicity of some IoT traffics. Even though time-varying Poisson processes are applied, the proposed algorithm can keep track of the actual number of backlogged users well. Although not shown here, we have studied via simulations the effect of the frequency in the cosine term, i.e., $a \cdot \cos(2\pi f t)$, to see how fast the proposed Bayesian algorithm can track the actual backlog size. We found out that the proposed algorithm could not work any more when $f \geq 1$. This is because, as the value of a becomes high, λ_t fluctuates widely. However, the value of f influences the algorithm more significantly.

V. Conclusion

This work has examined the stability of multichannel uplink NOMA RA systems. It has been proven that without controlling (re)transmissions from the backlogged users, the system always becomes unstable. In order to stabilize the system, we have proposed online retransmission control algorithm for the systems with two and three TRP levels, respectively, and analyzed the mean RA delay of the proposed algorithm. According to the proposed algorithm, the backlogged users will not retransmit if their SNR is too low to be above some threshold. Our analysis has provided an accurate performance measurement on the mean RA delay as the number of channels and that of TRP levels are raised and showed the invariance of the performance with respect to the threshold of SNR. We have demonstrated that the power consumption and RA delay can be reduced by introducing finer granularity in TRP levels.

As future work, we are interested in characterizing the convergence speed of the proposed algorithm explicitly. Further, it would be interesting to study how the proposed retransmission algorithm with channel inversion works under severe path loss so that the impact of users' location dependence can be captured. It is also worthwhile to incorporate the proposed algorithm into more advanced joint decoding techniques at the physical layer.

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