Social Welfare Maximization in Two-Tier Heterogeneous Cellular Networks

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***Abstract*—In this work, optimal bandwidth (BW) partitioning problem is studied for two-tier heterogeneous cellular networks with social welfare maximization (SWM) as an objective. Social welfare is represented as the sum of users' surplus and service provider's profit. For SWM, the unique optimal BW-fraction is analytically obtained for the two tiers, where spectrum is orthogonally partitioned between the tiers. To make the users indifferent to tiers for services, two different pricing schemes are presented. To ensure the feasibility of pricing schemes and uniqueness of optimal BW-fraction, limits on operational cost factor are defined. It is shown that, while satisfying the indifference principle, that provides equal surplus to the users from both tiers' services, the differential pricing scheme offers better control over the users' surplus and service provider's profit than the single pricing scheme. Simulations validate the analytical results. The effects of different system parameters on the optimal BW-fraction and the associated load to different tiers are also addressed.**

***Index Terms*—Heterogeneous cellular network, bandwidth partitioning, social welfare, pricing scheme, stochastic geometry.**

I. Introduction

Recently, heterogeneous cellular networks (HCN), wherein macro-cells are overlaid with small-cells, have emerged as effective means to enhance network capacity [1]. Small-cells are helpful in filling up the coverage holes and providing extra data service in hot-spots with relatively low capital expenditure (CAPEX) [2]. However, due to different cell densities and transmission power ranges in tiers, spectrum allocation, cross-tier interference management, and load balancing are critical issues in implementing HCN. In addition to this, for service provider (SP), network management in terms of pricing and resource allocation in different tiers has become complex.

Fractional frequency reuse [3] and almost-blank subframe technique [4] are well investigated for mitigation of cross-tier interference. However, these methods require proper coordination among different tiers, which becomes complex with large density of randomly deployed small-cells. Several studies have considered bandwidth (BW) partitioning as an effective solution to deal with the cross-tier interference [5]–[7]. In comparison to co-channel deployment (CCD), orthogonal spectrum deployment (OSD) scheme gives better results in terms of improved rate coverage [5], [6] and network utility [7]. In [5] it was shown that, for improving rate coverage, only load balancing is insufficient, BW partitioning is also required. However, optimal division of BW among the tiers was not defined. In [6], [7], the authors used flexible cell association to find optimal BW partitioning and biasing factor. However, flexible cell association scheme (biased maximum power based cell association scheme) is a load balancing scheme with CCD, which cannot be applied for BW partitioning with OSD. Note that, an association scheme defines load on different tiers, which in turn defines spectrum requirement in a tier to serve its associated users, and plays important role in BW partitioning among different tiers. In [9], millimeter wave based integrated-access-and-backhaul cellular network was considered. Optimal partitioning of the access and backhaul BW was obtained for small-cell base stations to maximize the rate coverage, and the same access BW was allocated to macro-cell. However, it was not considered how millimeter wave having limited signal transmission distance and small coverage range is suitable for macro-cell communication.

In [10], the authors developed a theoretical framework to compare the deployment cost of a cloud-based network against that of a traditional LTE network. However, the operational expenditure, pricing for services, social welfare, user's surplus, and SP's profit were not taken into account. In [11], [12], the economics aspects of multi-tier networks were investigated. In these papers, to maximize the SP's profit or social welfare, either BW partitioning is fixed [11] or associated users' densities are fixed [12]. However in our work, we fix neither BW partitioning nor associated users' density. Instead, we derive user's association probability to different tier using maximum signal-to-interference-plus-noise ratio (SINR) association scheme due to OSD, which defines the load on different tiers, and then we optimally divide the BW between tiers to maximize the social welfare. Also we see the effect of different system parameters on the associated load, optimal BW-fraction, user's surplus, SP's profit and social welfare.

Our main contribution is to construct an economic framework of two-tier HCNs based on social welfare maximization (SWM). The model enables us to obtain a unique optimal BW-fraction to maximize social welfare, when BW is orthogonally distributed over each tier. We additionally present two pricing schemes following indifference principle and show a condition of price per average data rate, under which users' surplus and SP's revenue can be positive.

II. System model

We consider a two-tier HCN, where macro- and small-cells are spatially distributed in a Euclidean space $\Re^2$ based on two independent homogeneous Poisson point processes (PPPs) Φ_m and Φ_s [13], with intensities γ_m and γ_s (cells/km^2), respectively, where m and s respectively stand for macro- and

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small-cells tier. Furthermore, users are distributed based on a PPP with intensity λ (users/km^2). We assume that macro- and small-cells employ downlink uniform transmission power spectral density (PSD) P_m and P_s, respectively, and a total of BW W is orthogonally split into two tiers. Frequency reuse factor for intra-tier cells is one. Thus, each macro- and small-cell has bandwidth $W_m = \zeta W$ and $W_s = (1-\zeta)W$, respectively, where $0 \leq \zeta \leq 1$ and $1-\zeta$ denote the normalized BW-fractions for macro- and small-cell tiers, respectively.

To begin with our economic framework, the surplus function of a user is defined as user's utility minus service cost that the user has to pay to the SP [15], [16]. Let d_k (bits/sec) be the average data rate to a user when it is associated with a cell of tier-k for $k \in \{m, s\}$, while $\mathcal{A}_k$ is the user's association probability with tier-k with the condition of it being within coverage of that tier, which is also used in defining the load on tier-k. Based on the above-stated definition of user's surplus function, the expected surplus U of a user is expressed as:

$$U = \sum_{k\in\{m,s\}} \mathcal{A}_k \left(\ln(1+d_k) - e_k d_k\right), \tag{1}$$

where e_k is price per unit data rate for the user associated with tier-k. Note that, we consider logarithmic function $U_t(d_k) = \ln(1+d_k)$ to characterize the user's utility $U_t(d_k)$. Such utility is commonly used to represent the diminishing return of getting additional data resource [14], [15]. Utility function of a user represents the degree of satisfaction that a user enjoys when achieving a specific data rate. Also, the considered utility function $U_t(d_k) = \ln(1+d_k)$ satisfies the following properties that a utility function should have:
1. $U_t(0) = 0$ and $U_t(d_k)$ is an increasing function of the allocated rate for a user.
2. Utility function is twice continuously differentiable in d_k.

To find $\mathcal{A}_k$ in (1), we first consider SINR of a user from a cell-i in tier-k, i.e.,

$$\mathrm{SINR}_{i,k} \triangleq \frac{P_k \cdot \omega_{k,i} h_i r_i^{-\alpha_k}}{\sum\limits_{n\in\Phi_k\setminus i} P_k \cdot \omega_{k,i} h_n {r_n}^{-\alpha_k} + \sigma^2 \cdot \omega_{k,i}}, \tag{2}$$

where $\omega_{k,i}$, h_i, and r_i are bandwidth assigned to the user, small-scale fading power gain, and distance of the tagged user from cell-i, respectively. We assume that the channels undergo statistically independent frequency non-selective Rayleigh fading, i.e., h_i is exponentially distributed with unit mean. $\alpha_k > 2$ and σ^2 are path loss factor and noise PSD, respectively. To obtain the net interference, the powers of all interfering signals are added that are received from each cell n belonging to PPP Φ_k except cell i of tier-k, i.e. $n \in \Phi_k\setminus i$, and the net interference term is expressed as: $\sum\limits_{n\in\phi_k\setminus i} P_k w_{k,i} h_n r_n^{-\alpha_k}$. Let Z_k be the maximum of SINRs that the tagged user has from a cell belonging to tier-k, i.e., $Z_k = \max_{i\in\Phi_k} \mathrm{SINR}_{i,k}$. Based on maximum SINR association rule, we express $\mathcal{A}_k$ in (1) as:

$$\begin{aligned}\mathcal{A}_k =& \Pr\left[Z_k \geq Z_l, Z_k \geq \theta_k, Z_l \geq \theta_l\right] \\ &+ \Pr[Z_k \geq \theta_k, Z_l \leq \theta_l].\end{aligned} \tag{3}$$

Here, if $k = m$, then $l = s$, or vice-versa. Note that θ_k is SINR threshold of tier-k. In (3), the first term on right-hand side (RHS) means that the maximum of SINRs of tier-k is larger than that of the other tier, while both are greater than or equal to the corresponding thresholds. The second term captures the case when only the maximum of SINRs of tier-k meets the threshold, i.e., user is under coverage of only one tier. Thus, the probability that a user is out of the coverage of both tiers is $1 - \mathcal{A}_s - \mathcal{A}_m$.

TABLE I
SUMMARY OF SYMBOLS

Symbol	Definition
γ_k	cells' density of tier-$k \in \{m, s\}$
λ	users density
P_k	uniform power spectral density of tier- k cell
W	total system bandwidth
ζ	normalized BW-fraction allocated to macro-cells tier
U	expected surplus of a user
$\mathcal{A}_k$	association probability of a user with tier-k
λ_k	users' density associated with tier-k
α_k	path loss exponent in tier-k
θ_k	SINR threshold for tier-k
T_k	average spectral efficiency in tier-k
d_k	data rate for a user when associated to tier-k
e_k	price per unit data rate for tier-k services
S_k	SP's profit from tier-k services
c	monetary cost per unit transmission power
$\mathcal{P}_{c_k}$	coverage probability due to tier-k

Appendix A examines $\mathcal{A}_k$ in (3).

Now, to find d_k in (1), let us denote by T_k (bits/sec/Hz) the average spectral efficiency of tier-k, which is examined in Appendix B. Since γ_k is the cells' density of tier-k, the total offered capacity of tier-k is $\gamma_k W_k T_k$, $k \in \{m, s\}$. If λ_k is the average density of users associated with tier-k, i.e., $\lambda_k = \lambda\mathcal{A}_k$, $d_k\lambda_k$ indicates the average data rate consumed by users of tier-k. Considering the capacity of tier-k equally distributed among the users in steady-state, we have $d_k\lambda_k = d_k\lambda\mathcal{A}_k = \gamma_k W_k T_k$, which yields $d_k = \gamma_k W_k T_k/(\lambda\mathcal{A}_k)$.

The profit of tier-k, S_k is expressed as revenue minus cost:

$$S_k = e_k\lambda_k d_k - c\gamma_k W_k P_k, \tag{4}$$

where c denotes operational expenditure (OPEX) factor also called as cost factor. It can be a monetary value per unit bandwidth and unit power. Price paid for service by users is revenue of the SP. Definitions of the symbols used in this paper are given in Table I.

III. Social Welfare Maximization with Pricing

This section considers the SWM problem and finds the limit on transmission cost c, to obtain a unique optimal BW-fraction. We subsequently discuss the pricing scheme and study their effects on the SP's profit and users' surplus.

The SWM problem of maximizing the sum of users' surplus and the SP's profit [16] is posed as:

$$\begin{aligned}&\underset{\zeta}{\text{maximize}} \quad \lambda U + \sum_{k\in\{m,s\}} S_k, \\ &\text{such that: } (C1): \quad 0 \leq \zeta \leq 1.\end{aligned} \tag{5}$$

Using (1) and (4), we rewrite (5) as

$$\begin{aligned}&\underset{\zeta}{\text{maximize}} \quad \sum_{k\in\{m,s\}} \left[\lambda_k \ln\left(1+d_k\right) - c\gamma_k W_k P_k\right] \\ &\text{such that:} \quad 0 \leq \zeta \leq 1.\end{aligned} \tag{6}$$

The following theorem shows unique ζ for maximizing (6).

Theorem 1: Optimal $\zeta^* \in [0,1]$ for SWM is obtained as

$$\zeta^* = \left(-B - \sqrt{B^2 - 4AC}\right)/(2A), \tag{7}$$

where $A = \chi \frac{\tau_s \tau_m}{\lambda_s \lambda_m}$, $\chi = cW(\gamma_m P_m - \gamma_s P_s)$, $\tau_m = \gamma_m W T_m$, $\tau_s = \gamma_s W T_s$, $B = -\chi \left(\frac{\tau_m}{\lambda_m}(1 + \frac{\tau_s}{\lambda_s}) - \frac{\tau_s}{\lambda_s}\right) - \tau_m \tau_s \left(\frac{1}{\lambda_m} + \frac{1}{\lambda_s}\right)$, and $C = (1 + \frac{\tau_s}{\lambda_s})(\tau_m - \chi) - \tau_s$.
For optimal ζ^* to be unique, the operational cost factor c should satisfy the following limits: for $\gamma_m P_m > \gamma_s P_s$:
$\max\left(0, \frac{\frac{\tau_m \lambda_m}{\tau_m + \lambda_m} - \tau_s}{W(\gamma_m P_m - \gamma_s P_s)}\right) < c < \frac{\tau_m - \frac{\tau_s \lambda_s}{\tau_s + \lambda_s}}{W(\gamma_m P_m - \gamma_s P_s)}$,
and for $\gamma_m P_m < \gamma_s P_s$:
$\max\left(0, \frac{\frac{\tau_s \lambda_s}{\tau_s + \lambda_s} - \tau_m}{W(\gamma_s P_s - \gamma_m P_m)}\right) < c < \frac{\tau_s - \frac{\tau_m \lambda_m}{\tau_m + \lambda_m}}{W(\gamma_s P_s - \gamma_m P_m)}$.

Proof: By substituting $d_m = \frac{\gamma_m W_m T_m}{\lambda_m} = \frac{\zeta \tau_m}{\lambda_m}$, $d_s = \frac{\gamma_s W_s T_s}{\lambda_s} = \frac{(1-\zeta)\tau_s}{\lambda_s}$, and $cW(\gamma_m P_m - \gamma_s P_s) = \chi$ in (6), we write social welfare $\mathcal{V}(\zeta)$ as follows:

$$\mathcal{V}(\zeta) = \lambda_m \ln\left(1 + \frac{\zeta \tau_m}{\lambda_m}\right) + \lambda_s \ln\left(1 + \frac{(1-\zeta)\tau_s}{\lambda_s}\right) - \zeta\chi - c\gamma_s W P_s. \tag{8}$$

From the 1st and 2nd derivative of $\mathcal{V}(\zeta)$ in (8) with respect to ζ, we get:

$$\frac{d\mathcal{V}(\zeta)}{d\zeta} = \frac{\lambda_m \tau_m}{(\zeta\tau_m + \lambda_m)} - \frac{\lambda_s \tau_s}{((1-\zeta)\tau_s + \lambda_s)} - \chi, \tag{9}$$

$$\frac{d^2\mathcal{V}(\zeta)}{d\zeta^2} = -\frac{\lambda_m \tau_m^2}{(\zeta\tau_m + \lambda_m)^2} - \frac{\lambda_s \tau_s^2}{((1-\zeta)\tau_s + \lambda_s)^2}. \tag{10}$$

Since $\frac{d^2\mathcal{V}(\zeta)}{d\zeta^2}$ in (10) is negative for all values of ζ, social welfare $\mathcal{V}(\zeta)$ is a concave function of BW-fraction ζ, and we obtain the optimal value of ζ to maximize social welfare $\mathcal{V}(\zeta)$ by equating $\frac{d\mathcal{V}(\zeta)}{d\zeta}$ in (9) to zero, which gives a quadratic equation as follows:

$$f(\zeta) = d\mathcal{V}(\zeta)/d\zeta = A\zeta^2 + B\zeta + C = 0, \tag{11}$$

where A, B, and C are given in Theorem 1. Condition for a unique root of this quadratic equation in $\zeta \in [0,1]$ is that $f(0) \cdot f(1) < 0$, which is expressed using (9) as:

$$\left(\tau_m - \chi - \frac{\tau_s \lambda_s}{\tau_s + \lambda_s}\right) \cdot \left(-\chi - \tau_s + \frac{\tau_m \lambda_m}{\tau_m + \lambda_m}\right) < 0. \tag{12}$$

This product in (12) will be negative if the two terms have different signs. Therefore, two cases can be considered. The first case is: $f(0) = \left(\tau_m - \chi - \frac{\tau_s \lambda_s}{\tau_s + \lambda_s}\right) < 0$ and
$f(1) = \left(-\chi - \tau_s + \frac{\tau_m \lambda_m}{\tau_m + \lambda_m}\right) > 0$.
Substituting $\chi = cW(\gamma_m P_m - \gamma_s P_s)$ and solving these conditions $f(0) < 0$ and $f(1) > 0$ for $\gamma_m P_m > \gamma_s P_s$ i.e. $A > 0$, we get range of cost factor c as:

$$\frac{\tau_m - \frac{\tau_s \lambda_s}{\tau_s + \lambda_s}}{W(\gamma_m P_m - \gamma_s P_s)} < c < \frac{\frac{\tau_m \lambda_m}{\tau_m + \lambda_m} - \tau_s}{W(\gamma_m P_m - \gamma_s P_s)}.$$

For the first case to be feasible, the lower limit of cost factor should be less than its upper limit, i.e., $\tau_m + \tau_s < \frac{\tau_m \lambda_m}{\tau_m + \lambda_m} + \frac{\tau_s \lambda_s}{\tau_s + \lambda_s}$, which is not possible due to value of both $\frac{\lambda_m}{\tau_m + \lambda_m}$ and $\frac{\lambda_s}{\tau_s + \lambda_s}$ being less than 1. Hence, the first case is not feasible.

Now, considering the second case: $f(0) = \left(\tau_m - \chi - \frac{\tau_s \lambda_s}{\tau_s + \lambda_s}\right) > 0$ and $f(1) = \left(-\chi - \tau_s + \frac{\tau_m \lambda_m}{\tau_m + \lambda_m}\right) < 0$, we get the following range of cost factor c as defined in Theorem 1:

$$\max\left(0, \frac{\frac{\tau_m \lambda_m}{\tau_m + \lambda_m} - \tau_s}{W(\gamma_m P_m - \gamma_s P_s)}\right) < c < \frac{\tau_m - \frac{\tau_s \lambda_s}{\tau_s + \lambda_s}}{W(\gamma_m P_m - \gamma_s P_s)}. \tag{13}$$

Note that, also for $\gamma_m P_m < \gamma_s P_s$ i.e. $A < 0$, the second case is feasible and the range of cost factor can be derived in similar manner.

Now, to get the formula of unique optimal BW-fraction, we use the second case ($f(0) > 0$ and $f(1) < 0$) that is feasible for providing the solution of $\frac{d\mathcal{V}(\zeta)}{d\zeta} = 0$ in (11) in terms of ζ in range $0 \le \zeta \le 1$. From the second case and (11), we have $f(0) = C > 0$ and $f(1) = A + B + C < 0$.

For $A > 0$ and $C > 0$, to satisfy $f(1) = A+B+C < 0$, B is < 0. With the conditions $A > 0$, $B < 0$ and $C > 0$, the unique optimal BW-fraction is represented as $\zeta^* = \frac{(-B - \sqrt{B^2 - 4AC})}{2A}$. The reason behind this is that for $A > 0$, $B < 0$ and $C > 0$, $\frac{(-B + \sqrt{B^2 - 4AC})}{2A} > \frac{(-B - \sqrt{B^2 - 4AC})}{2A} > 0$, and there can be only one solution between 0 and 1, therefore solution for ζ^* can not be $\frac{(-B + \sqrt{B^2 - 4AC})}{2A}$.

Now for $A < 0$ and $C > 0$, to satisfy $f(1) = A+B+C < 0$, B can be either $B > 0$ or $B < 0$. With $A < 0$ and $C > 0$, either B is positive or negative, the solution $\frac{(-B + \sqrt{B^2 - 4AC})}{2A}$ will provide a negative value. Hence, also for $A < 0$, the solution for ζ^* would be $\frac{(-B - \sqrt{B^2 - 4AC})}{2A}$. Thus, the unique optimal BW-fraction $\zeta^* \in [0,1]$ for SWM is obtained as:

$$\zeta^* = (-B - \sqrt{B^2 - 4AC})/(2A). \tag{14}$$

■

Remark: Theorem 1 defines the feasible range of cost factor c to get unique real-valued ζ^* in its valid range $[0,1]$ for SWM.

It is notable that, while the price e_k of tier k is taken into account in user's surplus in (1) and SP's profit in (4), it disappears in SWM (6). We determine e_k so as to make the user's surplus and SP's profit positive in following proposition.

Proposition 1: While guaranteeing the SP with profit S_k and providing the users with surplus U equally from both tiers, the price for users to pay should satisfy

$$\frac{S_k + c\gamma_k \zeta_k W P_k}{\lambda_k d_k} \le e_k \le \frac{\ln(1 + d_k) - U}{d_k}. \tag{15}$$

Proof: The first inequality is from $S_k = e_k \lambda_k d_k - c\gamma_k \zeta_k W P_k \ge 0$ in (4), where the second equality is obtained from $U = \ln(1 + d_k) - e_k d_k \ge 0$ in (1). ■

It is notable that in Proposition 1, as per the indifference principle, prices are set such that users get the same surplus U from each tier. In other words, they are indifferent to which tier they belong to such that they can agree with any value of $\mathcal{A}_k$ determined by (3), i.e., the maximum SINR association rule. If they would get a different surplus from each tier, the users

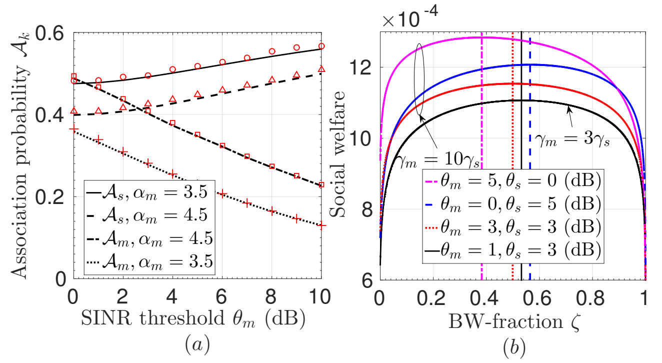

Fig. 1. (a) User association probabilities with macro- and small-cells; $\theta_s = 0$ dB, $\alpha_s = 4$, $\gamma_s = 3\gamma_m$. (b) Optimal BW-fraction for SWM with $\alpha_m = 4$, $\alpha_s = 4$, $W = 10$ KHz, $\lambda_u = 250$ users/Km^2.

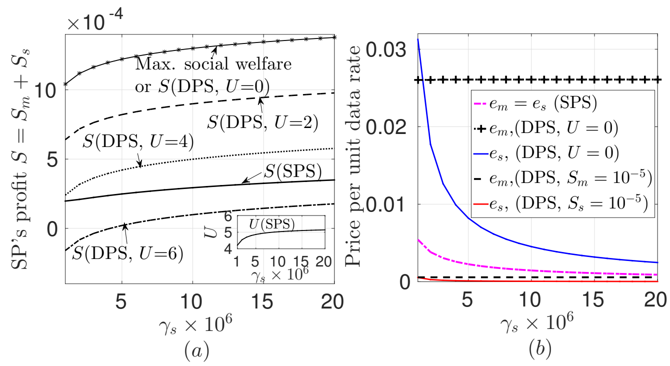

Fig. 2. (a) SP's profit with DPS for different μ, and fix profit and $\tilde{\mu}$ with SPS, with $\theta_m = 3$ dB, $\theta_s = 0$ dB (b) Price per unit data rate with SPS and DPS for different user's surplus and SP's profit.

would get a preference toward the tier that provides a higher surplus. It can be shown that if user's surplus U is too high to satisfy the inequality in (15), the network is in a deficit. Accordingly, (15) gives a feasible range of U and S_k. The numerator of the left-hand side (LHS) in (15) can correspond to gross revenue for the network, since it is the sum of profit margin and OPEX. The LHS shows that the gross revenue should be covered by associated users enjoying data rate.

If $e_s = e_m$, we can consider a single price scheme (SPS). Then, user's surplus U(SPS) and single price is determined as

$$U(\text{SPS}) = \ln\left(\frac{(1+d_s)^{d_m}}{(1+d_m)^{d_s}}\right)/(d_m - d_s), \tag{16}$$

$$e_s = e_m = \ln\left(\frac{1+d_m}{1+d_s}\right)/(d_m - d_s). \tag{17}$$

In SPS, user's surplus and SP's profit are fixed. However, if $e_m \neq e_s$, it is called a differential price scheme (DPS). Using DPS, SP can set the prices $e_m = \frac{\ln(1+d_m)-U}{d_m}$ and $e_s = \frac{\ln(1+d_s)-U}{d_s}$ to provide surplus U to all the users and accordingly SP can control its earned profit $S(\text{DPS}, U) = S_m + S_s$, where $S_k, k \in \{m, s\}$ is provided in (4). With DPS, the users can be made indifferent to being associated with any tier by providing them equal surplus U from both tiers' services. Note that, for a special case where $d_m = d_s = d$, only SPS can satisfy the indifference principle with $e_m = e_s = e < \frac{\ln(1+d)}{d}$ to provide positive surplus to the both tiers' users.

IV. Numerical studies

Fig. 1(a) validates the analysis of $\mathcal{A}_k, k \in \{m, s\}$ via simulations. For simulation, we set $\gamma_m = 1$, i.e., one macro-cell per km^2, $\gamma_s = 3$, $W = 10$MHz, $P_m W = 46$ (dBm), $P_s W = 23$ (dBm), $\sigma^2 = -150$ (dBm), $\alpha_s = 4$ and $\theta_s = 0$ (dB). To show the impact of path loss exponent and SINR thresholds on $\mathcal{A}_k$, we vary α_m and θ_m. Lines and markers in Fig. 1 (a) denote analysis and simulation, respectively. If θ_m is increased, more users are associated with small-cells tier due to increase of $\mathcal{A}_s$. On the other hand, for a higher α_m, $\mathcal{A}_m$ increases because the overall interference experienced by the users in macro-cells tier decreases.

Fig. 1(b) represents the optimal BW-fraction ζ^* from analysis (vertical lines) with SWM objective. It can be noticed that the maximum social welfare increases with the increase in γ_s. However, ζ^* decreases with increase in θ_m. Reduced ζ^* means lower BW-fraction for macro tier. With increase in θ_m the BW allocation to macro tier decreases, because with the higher θ_m, $\mathcal{A}_m$ decreases and higher number of users get associated to small-cells tier and to serve them higher resources are allocated to small-cells tier. One another important thing to notice is that for $\theta_m > \theta_s$, net social welfare is higher than the case where $\theta_m < \theta_s$, since the small-cells transmit at lower power in comparison of macro-cell and have lower transmit power cost, to improve social welfare.

Fig. 2 represents the SP's profit, user's surplus and service prices set with SPS and DPS. Here, we set OPEX factor monetary value, $c = 10^{-3}$, satisfying the range defined in Theorem 1. In Fig. 2(a), comparison of fix profit with SPS is shown with respect to the profit achievable with DPS at different surplus U of a user. It can be observed that, by controlling U, SP profit can be increased in DPS, while in SPS it is fixed. Fig. 2(b) is representing fix prices $e_m = e_s$ for SPS and upper and lower bounds (15) on e_k for $k \in \{m, s\}$ for DPS with $U = 0$ and $S_k = 10^{-5}$, respectively. For SPS, the fix possible SP's profit $S(\text{SPS})$ and the user's surplus $U(\text{SPS})$ are shown in main Fig. 2(a) and small figure inside Fig. 2(a), respectively. While using DPS, by setting different prices for both tier's service as shown in Fig. 2(b), SP can recover its loss in profit by controlling the user's surplus, and also can provide higher surplus to user by reducing the service prices even following indifference principle.

V. Conclusion

We studied bandwidth partitioning in two-tier HCN by estimating the unique BW-fraction for SWM. Pricing schemes following the indifference principle are considered to study the effect of service prices on users' surplus and SP's profit. Also we defined limits on operational cost factor providing unique ζ^*, and bounds on service prices such that users' surplus and SP's profit are positive. Considering maximum SINR association scheme, we analyzed user's association probabilities with different tiers. The effects of different system parameters, such as, α_m, α_s, θ_m, θ_s and γ_s were also addressed on $\mathcal{A}_k$ and ζ^*.

Appendix A
Association Probability $\mathcal{A}_k$

Before computing $\mathcal{A}_k$, Lemma 1 examines the coverage probability $\overline{\mathcal{P}}_{c_k} = \Pr[Z_k \geq \theta_k]$,which will be used in analysis of $\mathcal{A}_k$ and average spectral efficiency T_k (Appendix B).

Lemma 1: The coverage probability $\overline{\mathcal{P}}_{c_k}$ for $\theta_k > 1$ is

$$\overline{\mathcal{P}}_{c_k} = \Pr[Z_k \geq \theta_k] = 2\pi\gamma_k \int_0^\infty r\, g_k(r,\theta_k)dr. \quad (18)$$

where $g_k(r,x) = \exp\left(\frac{-\pi\gamma_k\eta(\alpha_k)r^2}{x^{-2/\alpha_k}}\right) \cdot \exp\left(\frac{-x\sigma^2 r^{\alpha_k}}{P_k}\right)$, and $\eta(\alpha_k) = \int\limits_0^\infty \frac{1}{\left(1+t^{\frac{\alpha_k}{2}}\right)} dt.$

Proof: Keeping in mind that $\overline{\mathcal{P}}_{c_k}$ due to both tiers are separately obtained and there is no inter-tier interference due to OSD, following Theorem 1 in [8], we have (18). ■

The following proposition derives the association probabilities $\mathcal{A}_k, k \in \{m,s\}$, of a tagged user with both tiers.

Proposition 2: $\mathcal{A}_k, k \in \{m,s\}$, are obtained as:

$$\mathcal{A}_k = \overline{\mathcal{P}}_{c_k} - \int_{\theta_k}^\infty f_{Z_k}(z_k)\left(2\pi\gamma_l \int_0^\infty r\, g_l(r,z_k)dr\right)dz_k \quad (19)$$

and

$$\mathcal{A}_l = \overline{\mathcal{P}}_{c_k}(1-F_{Z_l}(\theta_k)) + (1-\overline{\mathcal{P}}_{c_k})\overline{\mathcal{P}}_{c_l} - \int_{\theta_k}^\infty f_{Z_l}(z_l)\left(2\pi\gamma_k \int_0^\infty r\, g_k(r,z_l)dr\right)dz_l. \quad (20)$$

Here, if $k=m$, $l=s$, and vice-versa having $\theta_k > \theta_l$.

Proof: To get $\mathcal{A}_k$, we consider Z_k as a random variable, which is greater than one. Using (18), its probability distribution function (PDF) $f_{Z_k}(z_k)$ and cumulative distribution function (CDF) $F_{Z_k}(z_k)$ respectively are written as:

$$f_{Z_k}(z_k) = \frac{2\pi\gamma_k\sigma^2}{P_k}\int_0^\infty r^{(\alpha_k+1)} g_k(r,z_k)dr + \frac{4\pi^2\gamma_k^2\eta(\alpha_k)}{\alpha_k z_k^{-2/\alpha_k+1}}\int_0^\infty r^3 g_k(r,z_k)dr, \quad (21)$$

$$F_{Z_k}(z_k) = 1 - 2\pi\gamma_k\int_0^\infty r\, g_k(r,z_k)dr. \quad (22)$$

Since PDFs of Z_k and Z_l are independent, to solve $\mathcal{A}_k$ (3) for $\theta_k > \theta_l$, we have:

$$\begin{aligned}
&\Pr\left[Z_k > Z_l, Z_k > \theta_k, Z_l > \theta_l\right] \\
&= \int_{\theta_k}^\infty f_{Z_k}(z_k)\left(\int_{\theta_l}^{z_k} f_{Z_l}(z_l)dz_l\right)dz_k \\
&= \int_{\theta_k}^\infty f_{Z_k}(z_k)(F_{Z_l}(z_k) - F_{Z_l}(\theta_l))dz_k \\
&= \int_{\theta_k}^\infty f_{Z_k}(z_k)F_{Z_l}(z_k)dz_k - (1-\overline{\mathcal{P}}_{c_l})\overline{\mathcal{P}}_{c_k}. \quad (23)
\end{aligned}$$

Substituting this and $\Pr[Z_k > \theta_k, Z_l < \theta_l] = \overline{\mathcal{P}}_{c_k}(1-\overline{\mathcal{P}}_{c_l})$ in (3) gives $\mathcal{A}_k$ in (19). To find $\mathcal{A}_l$, for $\theta_k > \theta_l$, we have:

$$\begin{aligned}
&\Pr\left[Z_l > Z_k, Z_k > \theta_k, Z_l > \theta_l\right] \\
&= \int_{\theta_k}^\infty f_{Z_l}(z_l)\left(\int_{\theta_k}^{z_l} f_{Z_k}(z_k)dz_k\right)dz_l \\
&= \int_{\theta_k}^\infty f_{Z_l}(z_l)(F_{Z_k}(z_l) - F_{Z_k}(\theta_k))dz_l \\
&= \int_{\theta_k}^\infty f_{Z_l}(z_l)F_{Z_k}(z_l)dz_l - (1-\overline{\mathcal{P}}_{c_k})(1-F_{Z_l}(\theta_k)). \quad (24)
\end{aligned}$$

Using (24) and $\Pr[Z_k < \theta_k, Z_l > \theta_l] = \overline{\mathcal{P}}_{c_l}(1-\overline{\mathcal{P}}_{c_k})$, we get $\mathcal{A}_l$ defined in (20). ■

Appendix B
Average spectral efficiency T_k

T_k denotes the average spectral efficiency achievable by a random tagged user within the coverage of tier-k, for $k \in \{m,s\}$, and can be expressed as:

$$T_k = \mathbb{E}\left[\log_2(1+Z_k)|Z_k \geq \theta_k\right]. \quad (25)$$

Using Lemma 1, we have:

$$1 - F_{Z_k}(z|\theta_k) = \Pr[Z_k \geq z | Z_k \geq \theta_k] = \begin{cases} \frac{2\pi\gamma_k}{\overline{\mathcal{P}}_{c_k}}\int_0^\infty r g_k(r,z)dr, & \text{if } z > \theta_k; \\ 1, & \text{otherwise.} \end{cases}$$

Further solving T_k (25) as in Theorem 2 in [8], we get:

$$T_k = \log_2(1+\theta_k) + \frac{1}{(\overline{\mathcal{P}}_{c_k}\cdot \ln 2)} \times \int_{\theta_k}^\infty \frac{2\pi\gamma_k}{1+y}\int_0^\infty r\, g_k(r,y)drdy. \quad (26)$$

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