Optimal Time Allocation for RF-powered DF Relay-assisted Cooperative Communication

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In this letter, we consider a decode-and-forward (DF) relay-assisted RF-powered cooperative communication between an RF energy harvesting source and an information sink. DF relay aids the communication between these two distant wireless nodes by (i) RF energy transfer to the source, and (ii) relaying source data to the sink. To enable efficient RF-powered delay-constrained information transfer to the sink under Rayleigh fading, joint global-optimal time allocations for information and energy transfer are derived. Impact of relay position on throughput and optimality of the analytical solutions are numerically investigated.

Introduction: With the advent of 5G and need for connecting exponentially growing wireless devices in Internet of Things (IoT), controlled energy replenishment via dedicated Radio Frequency (RF) Energy Transfer (RFET) has emerged as a promising technique to realize uninterrupted network operation [1]. Due to dual usage of same RF signal for RFET and wireless information transfer (WIT), cooperative relaying techniques have been recently studied [1]–[9] to overcome the large gap in their respective reception sensitivities (≈ -10 dBm for RFET and ≈ -60 dBm for WIT) and hence enable efficient joint RFET and WIT.

As cooperative communication techniques for joint WIT and RFET require a very different paradigm as compared to WIT alone [1], the existing research works either considered RF energy harvesting (RFH) relays [2, 3, 4], RFH destinations [2, 5], hybrid access point powered communication [6], or the usage of power beacon for powering source and relay [7]. More recently, decode-and-forward (DF) relay-powered cooperative communication with RFH source was considered in [8, 9].

We consider a two-hop half-duplex DF relaying system, with energy-constrained RFH source $\mathcal{S}$ that communicates to destination $\mathcal{D}$ via DF relay $\mathcal{R}$. This scenario is highly relevant in context of battery-constrained information sources like RFH sensors and low-power wireless devices in IoT, which can be assisted by $\mathcal{R}$ through on-demand energy replenishment and information relaying. Different from [8, 9], which studied maximization of *delay-tolerant data rate without considering the effect of channel randomness on harvested energy* at $\mathcal{S}$, in this letter *optimal time allocation (OTA)* is derived for RFET and WIT to maximize *delay-constrained throughput under Rayleigh fading channel*. Note that, [8] considered fixed time allocation and [9] derived numerical solutions, whereas we have additionally derived the *analytical solutions for OTA*.

System model: The considered system model is shown in Fig. 1, where an RFH $\mathcal{S}$, powered by an energy-rich DF information relay $\mathcal{R}$, communicates with $\mathcal{D}$ in two-hop fashion via $\mathcal{R}$. So, $\mathcal{R}$, acting as both energy source and information relay for $\mathcal{S}$, can be considered as a power beacon [7] with additional data processing and transmission ability. $\mathcal{S}$ and $\mathcal{D}$ are equipped with single antenna, whereas $\mathcal{R}$ has two antennas, respectively directed towards $\mathcal{S}$ and $\mathcal{D}$ for efficient RFET and WIT. $\mathcal{S}$ has an RFH unit and supercapacitor for storing the energy received from $\mathcal{R}$. $\mathcal{S}$-to-$\mathcal{D}$ direct communication link is assumed unavailable due to large path loss and blockage by obstacles. The channels: $\mathcal{R}$-to-$\mathcal{S}$ (h_0), $\mathcal{S}$-to-$\mathcal{R}$ (h_1), and $\mathcal{R}$-to-$\mathcal{D}$ (h_2) are assumed statistically independent, with frequency non-selective Rayleigh block fading, having average powers $\mathbb{E}\left[|h_0|^2\right] = \frac{a_0}{d_1^\alpha}$, $\mathbb{E}\left[|h_1|^2\right] = \frac{a_1}{d_1^\alpha}$, and $\mathbb{E}\left[|h_2|^2\right] = \frac{a_2}{d_2^\alpha}$. Here, α is path loss exponent; a_0, a_1, and a_2 respectively account for different $\mathcal{R}$-to-$\mathcal{S}$, $\mathcal{S}$-to-$\mathcal{R}$, and $\mathcal{R}$-to-$\mathcal{D}$ channel gains; d_1 and d_2 are $\mathcal{S}$-to-$\mathcal{R}$ and $\mathcal{R}$-to-$\mathcal{D}$ distances. It is assumed that channel reciprocity holds in $\mathcal{S}$-to-$\mathcal{R}$ link with $|h_0|^2 = |h_1|^2$ and the channel state information is available at $\mathcal{R}$ and $\mathcal{D}$.

Considering a block duration of T sec, proposed relay-assisted cooperative communication system comprises of three phases (Fig. 1). Phase 1 of $\rho_e T$ duration for RFET from $\mathcal{S}$-to-$\mathcal{R}$ to enable information transfer operation at $\mathcal{S}$. Phase 2 of $\rho_i T$ duration is dedicated for WIT from $\mathcal{S}$-to-$\mathcal{R}$ using energy harvested during $\rho_e T$. Finally, $\mathcal{R}$ forwards the decoded information signal received from $\mathcal{S}$ to $\mathcal{D}$ in Phase 3 of duration $\rho_i T$. Thus, intuitively $\rho_i = \frac{1-\rho_e}{2}$. Considering that the strength of noise power at $\mathcal{S}$ is negligible as compared to the received signal power from $\mathcal{R}$, the amount of energy $E_{\mathcal{S}}$ harvested at $\mathcal{S}$ during $\rho_e T$ is given by:

$$E_{\mathcal{S}} = \eta_{\mathcal{S}} P_{\mathcal{R}} \, |h_0|^2 \, \rho_e T \tag{1}$$

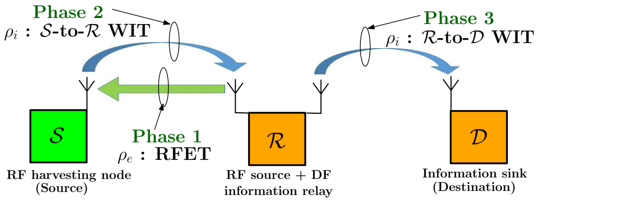


Fig. 1. *DF relay assisted RF-powered cooperative communication.*

where $\eta_{\mathcal{S}}$ is RF-to-DC rectification efficiency of RFH unit and $P_{\mathcal{R}}$ is transmit power of $\mathcal{R}$. Using $E_{\mathcal{S}}$, $\mathcal{S}$ transmits at power $P_{\mathcal{S}} = \frac{E_{\mathcal{S}}}{\rho_i T}$ for WIT to $\mathcal{D}$ via $\mathcal{R}$. The signal received at $\mathcal{R}$ from $\mathcal{S}$ is $y_{i_{\mathcal{R}}} = h_1 \sqrt{P_{\mathcal{S}}} x_{i_{\mathcal{S}}} + \aleph_{\mathcal{R}}$, where $x_{i_{\mathcal{S}}}$ is the normalized information symbol transmitted by $\mathcal{S}$ having zero mean and unit variance. $\aleph_{\mathcal{R}}$ is Additive White Gaussian Noise (AWGN) at $\mathcal{R}$. On receiving $y_{i_{\mathcal{R}}}$ from $\mathcal{S}$, $\mathcal{R}$ decodes it and forwards the decoded unit power signal $\widehat{x_{i_{\mathcal{S}}}}$ to $\mathcal{D}$ in next slot (Phase 3). RF signal received at $\mathcal{D}$ is: $y_{i_{\mathcal{D}}} = h_2 \sqrt{P_{\mathcal{R}}} \widehat{x_{i_{\mathcal{S}}}} + \aleph_{\mathcal{D}}$, where $\aleph_{\mathcal{D}}$ is AWGN at $\mathcal{D}$. $\aleph_{\mathcal{R}}$ and $\aleph_{\mathcal{D}}$ are mutually independent with zero mean and variance σ^2.

In our considered delay-constrained relay-powered WIT, we next derive the outage probability for achieving a desired $\mathcal{S}$-to-$\mathcal{D}$ data rate.

Outage analysis: Without $\mathcal{S}$-to-$\mathcal{D}$ direct link, end-to-end signal-to-noise ratio (SNR) γ_{E2E} is limited by the lower value between $\mathcal{S}$-to-$\mathcal{R}$ SNR γ_1 and $\mathcal{R}$-to-$\mathcal{D}$ SNR γ_2, which are assumed independent. Outage probability p_{out}, defined under path loss and Rayleigh fading, is the probability that received data rate at $\mathcal{D}$ falls below a threshold λ_m. Mathematically,

$$\begin{aligned} p_{out} &= \Pr\left[\log_2(1+\gamma_{E2E}) < \lambda_m\right] = \Pr\left[\min\{\gamma_1, \gamma_2\} < 2^{\lambda_m} - 1\right] \\ &= 1 - \left(\Pr\left[\gamma_1 > 2^{\lambda_m} - 1\right]\right)\left(\Pr\left[\gamma_2 > 2^{\lambda_m} - 1\right]\right). \end{aligned} \tag{2}$$

For Rayleigh fading, γ_2 is exponentially distributed with tail distribution:

$$\Pr\left[\gamma_2 > \Lambda\right] = e^{-\Upsilon_b \Lambda}, \quad \text{where } \Upsilon_b \triangleq \frac{(d_2)^\alpha \sigma^2}{a_2 P_{\mathcal{R}}} \text{ and } \Lambda \triangleq 2^{\lambda_m} - 1. \tag{3}$$

Tail distribution of γ_1, which involves product of two independent and identically distributed exponential random variables $|h_0|^2 = |h_1|^2$, is:

$$\Pr\left[\gamma_1 > \Lambda\right] = \Upsilon_a \int_{x=0}^{\infty} e^{-\Upsilon_a\left(x + \frac{\Lambda}{x}\right)} dx = 2\Upsilon_a \sqrt{\Lambda}\, \mathbf{K_1}\left(2\Upsilon_a\sqrt{\Lambda}\right). \tag{4}$$

Here $\Upsilon_a \triangleq \sqrt{\frac{\sigma^2(1-\rho_e)(d_1)^{2\alpha}}{2\eta_{\mathcal{S}} a_1^2 P_{\mathcal{R}} \rho_e}}$ and $\mathbf{K_n}(\cdot)$ is the nth order modified Bessel function of second kind. Using (3) and (4), p_{out} in (2) can be obtained.

Throughput maximization: The achievable throughput in a delay-constrained scenario is $\tau = \rho_i \lambda_m (1 - p_{out})$, which depends on the source transmission rate λ_m and p_{out} in achieving this rate at $\mathcal{D}$. With τ (and p_{out}) as a function of time allocation ρ_e for RFET and ρ_i for WIT operations, for typical inter-node distances d_1, d_2, and relay transmit power $P_{\mathcal{R}}$, we intend to find the OTA ρ_e^* and ρ_i^* that maximize τ. With $\rho_i = \frac{1-\rho_e}{2}$, Throughput Maximization Problem (TMP) is defined as:

$$\begin{aligned} \textbf{TMP}: \quad & \underset{\rho_e}{\text{maximize}} \quad \tau = (1-\rho_e)\,\lambda_m \Upsilon_a \sqrt{\Lambda}\, \mathbf{K_1}\left(2\Upsilon_a\sqrt{\Lambda}\right) e^{-\Upsilon_b \Lambda} \\ & \text{subject to} \quad C1: \rho_e \le 1, \qquad C2: \rho_e \ge 0. \end{aligned} \tag{5}$$

Note that $\tau = 0$ for $\rho_e = 0 \vee \rho_e = 1$. So for positive τ, $0 < \rho_e < 1$ and the Lagrange multipliers [10] corresponding to constraints $C1$ and $C2$ are zero. Hence, the Karush-Kuhn-Tucker (KKT) point ρ_e^* of TMP is obtained by solving the following KKT stationarity condition.

$$\frac{\partial \tau}{\partial \rho_e} = \frac{\mathcal{C}_2\sqrt{\mathfrak{C}}}{4\rho_e}\left(\sqrt{\mathfrak{C}}\, \mathbf{K_0}\left(\sqrt{\mathfrak{C}}\right) - 2\rho_e \mathbf{K_1}\left(\sqrt{\mathfrak{C}}\right)\right) = 0 \tag{6}$$

where $\mathfrak{C} \triangleq \frac{\mathcal{C}_1(1-\rho_e)}{2\rho_e}$ with $\mathcal{C}_1 \triangleq \frac{4\sigma^2(d_1)^{2\alpha}\Lambda}{\eta_{\mathcal{S}} a_1^2 P_{\mathcal{R}}}$, and $\mathcal{C}_2 \triangleq \lambda_m e^{-\Upsilon_b \Lambda}$. As $\mathcal{C}_2 \neq 0$, $\mathfrak{C} \neq 0$, optimal ρ_e^* is obtained by solving the following equation:

$$\sqrt{\mathfrak{C}}\, \mathbf{K_0}\left(\sqrt{\mathfrak{C}}\right) = 2\rho_e \mathbf{K_1}\left(\sqrt{\mathfrak{C}}\right). \tag{7}$$

Although it is not possible to analytically solve this equation, numerical solution for ρ_e^* can be obtained using commercial numerical solvers or root-finding schemes. Using this numerical solution ρ_e^*, global-optimal $\rho_i^* = \frac{1-\rho_e^*}{2}$ can be obtained. Optimality of the derived ρ_e^* and ρ_i^* can be proved using the proposed Theorem 1 and results in [10, Theorem 4.3.8].

Theorem 1: The objective function τ to be maximized in TMP is a concave function of ρ_e^*, the constraint functions $C1$ and $C2$ are differentiable and affine (or convex) functions of ρ_e^*, and the KKT conditions hold at ρ_e^* with Lagrange multipliers for $C1$ and $C2$ as zero.

Proof: The Hessian or second derivative of τ with respect to ρ_e is:

$$\frac{\partial^2\tau}{\partial\rho_e^2} = -\frac{\mathcal{C}_1\mathcal{C}_2}{16\rho_e^3}\left(4\,\mathbf{K_0}\left(\sqrt{\mathfrak{C}}\right) - \sqrt{\mathfrak{C}}\,\mathbf{K_1}\left(\sqrt{\mathfrak{C}}\right)\right). \quad (8)$$

For proving concavity of τ in ρ_e, it suffices to show that $\frac{\partial^2\tau}{\partial\rho_e^2} \le 0$. With $\mathcal{C}_1, \mathcal{C}_2, \rho_e \ge 0$, from (8) the non-positivity of $\frac{\partial^2\tau}{\partial\rho_e^2}$ simply requires:

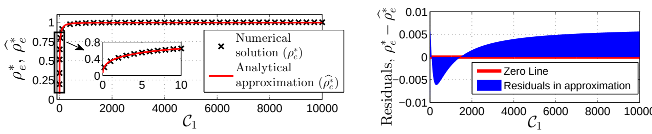
$$4\mathbf{K_0}\left(\sqrt{\mathfrak{C}}\right) \ge \sqrt{\mathfrak{C}}\,\mathbf{K_1}\left(\sqrt{\mathfrak{C}}\right), \quad \text{or on simplification} \quad \mathfrak{C} < 3.52827. \quad (9)$$

Since $\mathcal{C}_1 \ge 0$ and $0 < \rho_e < 1$, ρ_e^* obtained by solving (7) provides bounds on $\mathfrak{C}$ for the KKT conditions to hold at ρ_e^*, which are given by: $0 \le \mathfrak{C} \le 2.3867$. This condition together with (9) shows that the maximum achievable throughput τ^* is a concave function of a feasible KKT point ($0 < \rho_e^* < 1$). This along with convexity of $C1$–$C2$ and [10, Theorem 4.3.8], shows that ρ_e^* is the global-optimal solution of TMP. ■

Proposed tight analytical approximation for ρ_e^*: To gain analytical insights on optimal ρ_e^* and ρ_i^*, and interplay between the influential system parameters, we propose a closed-form solution for ρ_e^* and ρ_i^* by exploiting the numerical relationship between ρ_e^* and $\mathcal{C}_1$. Using $\mathfrak{C} = \frac{\mathcal{C}_1(1-\rho_e)}{2\rho_e}$, we plot the numerical solution ρ_e^* of (7) with varying $\mathcal{C}_1$ in Fig. 2(a). A tight exponential approximation $\widehat{\rho_e^*}$ for ρ_e^* to capture this relationship is given by the following analytical expression:

$$\widehat{\rho_e^*} \triangleq 0.9932 - \exp\left(-e^{-0.8275}\left(\frac{4\sigma^2 d_1^{2\alpha}\left(2^{\lambda_m}-1\right)}{\eta_{\mathcal{S}}\, a_1^2 P_{\mathcal{R}}}\right)^{0.3808}\right). \quad (10)$$

Using $\widehat{\rho_e^*}$, analytical solution for ρ_i^* is: $\widehat{\rho_i^*} = \frac{1-\widehat{\rho_e^*}}{2}$. Analytical $\widehat{\rho_e^*}$ very closely follows numerical ρ_e^*, as shown in Fig. 2(b) with the help of residuals plot. The root mean square error value 0.0042 (very close to 0) and R-square statistics value 0.9997 (very close to 1) signify the goodness of proposed exponential fit. Throughput performance with $\widehat{\rho_e^*}$, providing insights on role of parameters like d_1, λ_m, and $P_{\mathcal{R}}$, is very similar to that with global-optimal ρ_e^*, as discussed in next section (cf. Fig. 4(b)).



(a) Finding analytical relationship (b) Residuals plot

Fig. 2. *Validation of the proposed analytical approximation for ρ_e^*.*

Numerical results: For performance evaluation of proposed TMP, the following parameters are considered: $a_0 = a_1 = a_2 = 1$; $\alpha = 3$; $\lambda_m = 10$ bps/Hz; $P_{\mathcal{R}} = 10$ W; $\sigma^2 = 10^{-13}$; $\eta_{\mathcal{S}} = 0.8$; and $T = 1$ sec. $\mathcal{R}$ is located such that $d_1 + d_2 =$ constant (100 m in our experiments). (The position of $\mathcal{R}$ is not necessarily on line joining $\mathcal{S}$ and $\mathcal{D}$). So, $d_2 = 100 - d_1$.

The variation of τ with ρ_e and ρ_i is plotted in Fig. 3 for different $\mathcal{S}$-to-$\mathcal{R}$ distances d_1. The results show that, a higher throughput is achieved when $\mathcal{R}$ is closer to $\mathcal{S}$. The maximum achievable throughput τ^* decreases with increased d_1. This is due to the doubly-near-far problem faced by relay-powered source communication, where the mean received power at $\mathcal{R}$, $\mathbb{E}\left[P_{\mathcal{R}}^r\right] = \frac{2\eta_{\mathcal{S}} a_1^2 P_{\mathcal{R}} \rho_e}{(1-\rho_e)(d_1)^{2\alpha}}$, i.e., $\mathbb{E}\left[P_{\mathcal{R}}^r\right] \propto (d_1)^{-2\alpha}$. Also, analytical OTA $\widehat{\rho_e^*}$ and $\widehat{\rho_i^*}$, respectively increases and decreases with increased d_1.

We next investigate optimal position of $\mathcal{R}$ to maximize the throughput efficiency of cooperative RF-powered communication. Variation of OTA and τ^* with increase in d_1 is shown in Fig. 4. It is observed that the optimal position of $\mathcal{R}$ is close to $\mathcal{S}$ because, as $\mathcal{R}$ is moved away from $\mathcal{S}$ more RFET time is required, which leads to lower ρ_i^*, and thus lower τ^*. Results plotted in Fig. 4(a) also show that the analytical approximations for OTA $\widehat{\rho_e^*}$ and $\widehat{\rho_i^*}$ closely follow the numerical OTA ρ_e^* and ρ_i^*, with a minor deterioration of less than 0.0281% in throughput performance.

We have also compared the throughput performance of the proposed OTA ρ_e^* and ρ_i^* with the benchmark uniform allocation scheme having $\rho_e = \rho_i = \frac{1}{3}$. Fig. 4(b) shows that the proposed OTA outperforms the

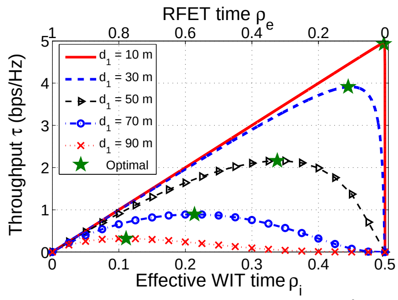
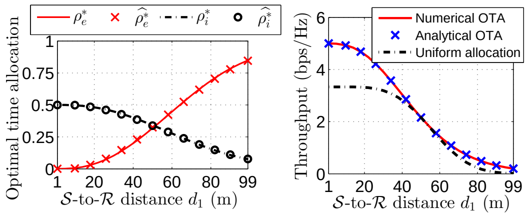


Fig. 3. *Variation of τ and analytical OTA ($\widehat{\rho_e^*}$ and $\widehat{\rho_i^*}$) for different d_1.*



(a) OTA (numerical and analytical) (b) Achievable thoughput comparison

Fig. 4. *Performance comparison with benchmark scheme for varying d_1.*

uniform allocation scheme in terms of significantly enhanced throughput, with an average increase of 166.11% for varying d_1, corroborating the importance of the proposed OTA in RF-powered communications.

Concluding remarks: We considered DF relay-powered two-hop WIT between an RFH source $\mathcal{S}$ and information sink $\mathcal{D}$. To enable efficient RF-powered delay-constrained WIT, we first derived outage probability at $\mathcal{D}$ for achieving a desired rate. After that we proved global-optimality of throughput maximization problem and obtained OTA for RFET and WIT. We also derived closed-form expression for tight approximation of OTA to gain analytical insights on the role of different system parameters. The numerical results offered insights about the optimal relay position. Finally, we showed that the proposed OTA offers significant throughput enhancement over uniform allocation, which can help in realization of perpetual operation of RF-powered communication networks.

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