Low Complexity Dimensioning of Sustainable Solar-enabled Systems: A Case of Base Station

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Abstract—Solar-enabled systems are becoming popular for provisioning pollution-free and cost-effective energy solution. Dimensioning of a solar-enabled system requires estimation of appropriate size of photovoltaic (PV) panel as well as storage capacity while satisfying a given energy outage constraint. Dimensioning has strong impact on the user's quality of experience and network operator's interest in terms of energy outage and revenue. In this paper, dimensioning problem of solar-enabled communication nodes is analyzed in order to reduce the computation overhead, where stand-alone solar-enabled base station (SS-BS) is considered as a case study. For this purpose, hourly solar data of last ten years has been taken into consideration for analysis. First, the power consumption model of BS is revised to save energy and increase revenue. Using the hourly solar data and power consumption profile, the lower bounds on panel size and storage capacity are obtained using Gaussian mixture model, which provides a reduced search space for cost-optimal system dimensioning. Then, the cost function and energy outage probability are modeled as functions of panel size and number of battery units using curve fitting technique. The cost function is proven to be quasiconvex, whereas energy outage probability is proven to be convex function of panel size and number of battery units. These properties transform the cost-optimal dimensioning problem into a convex optimization framework, which ensures a global optimal solution. Finally, a Computationally-efficient Energy outage aware Cost-optimal Dimensioning Algorithm (CECoDA) is proposed to estimate the system dimension without requiring exhaustive search. The proposed framework is tested and validated on solar data of several cities; for illustration purpose, four cities, New Delhi, Itanagar, Las Vegas, and Kansas, located at diverse geographical regions, are considered. It is demonstrated that, the presented optimization framework determines the system dimension accurately, while reducing the computational overhead up to 94% and the associated energy requirement for computation with respect to the exhaustive search method used in the existing approaches. The proposed framework CECoDA takes advantage of the location-dependent unique solar profile, thereby achieving cost-efficient solar-enabled system design in significantly less time.

Index Terms—Sustainable solar-enabled system, solar energy harvesting, cost-optimal system dimensioning, energy outage, Gaussian mixture model, curve fitting, convex optimization, computation efficiency

1 INTRODUCTION

Renewable sources of energy, such as solar and wind, are considered to be promising alternatives for powering the wireless nodes for sensing, communication, and processing activities in sensor networks [1], agricultural systems [2], home appliances [3], cellular base stations (BSs) [4], and data centers [5]. Environmentally friendly, cost-effective, and less maintenance requirement are some key features, which encourage to adopt the renewable energy based solution. Easy sunlight availability at most of the places, less costly, and noiseless operation are some important attributes, which suggest preference of solar over wind energy. Solar energy is beneficial in reducing greenhouse gas emission associated with electricity generation and achieving energy security.

A solar-enabled system requires a suitably-sized solar-energy harvesting panel and an appropriate energy storage capacity for its uninterrupted operation in absence of sufficient harvested energy. These two dimensioning requirements are interdependent. An optimal dimensioning should ensure a minimum energy outage and system cost while dealing with load profile that could be of stochastic nature. The energy harvesting and storage dimensioning principles are broadly the same for various applications, such as in wireless access infrastructure and sensor networks. In this work, we focus on stand-alone solar-enabled BSs (SS-BSs), where energy requirement profile for BS is highly time-varying due to the nature of communication traffic that the BS has to serve over 24 hours of a day.

Motivation behind our consideration of SS-BS dimensioning lies in its complexity and more generalized system configuration in terms of very long-term operation (typically, 10-15 years), replacement cost of system components (e.g., battery), and real-time traffic handling while satisfying stringent quality of service (QoS) requirements. Therefore, the analysis of cost-optimal SS-BS dimensioning covers the dimensioning related analysis of other solar-enabled systems, such as, water-pumping system, home energy, data center, sensor network, etc. Furthermore, there are a large number of geographical pockets especially in the developing countries, where the cellular BSs are not connected to power grid. In such cases, a typical BS consumes about 1500 liters of diesel per month [6]. Hence, cost-optimal energy system design of SS-BS is expected to have a high impact in realization of green communications.

The energy consumption in information and communications technology infrastructure provisioning is becoming a severe issue in terms of economy as well as environment. To address these issues, SS-BS has been gaining significant interest and is being deployed worldwide to mitigate this issue. While dimensioning the SS-BS, it requires to process very large volume of solar data for accurate dimensioning of the system, which leads to very high computation overhead and hence energy consumption. Therefore,
design of algorithms for optimally reducing the computation overhead in SS-BS dimensioning is of great importance.

2 BACKGROUND AND MOTIVATION

SS-BSs are equipped with photovoltaic (PV) panel and storage device, i.e., rechargeable battery. As the energy is harvested during the day-time only, the excess energy is required to be stored in the battery for night-time operation when no energy harvesting takes place. In SS-BSs, it is important to accurately dimension the required panel size and the number of battery units for sustainable operation in the long run while minimizing energy outage. Energy outage refers to an event when the energy required by the load exceeds the available energy in the system, and as a result routine activities of the wireless node are interrupted. Dimensioning has significant importance in terms of QoS as well as quality-of-experience (QoE) of the users’ perspective - that seek minimum energy blackout, and economics from network operator’s perspective - that wishes to have minimum investment and recurring costs to achieve revenue. The investment in the given context comprises capital expenditure (CapEx), operational expenditure (OpEx), and implementation expenditure (ImpEx). CapEx accounts for initial investment, OpEx accounts for the cost of replacement of system components, whereas ImpEx accounts for the rental cost. Rental cost refers to the rent of SS-BS deployment site, where cell tower, control room, and array of solar panels are being installed [7]. The solar panels cover significant space, for example, a reference solar panel of size 1 KW covers approximately 5 m² area. Hence, the space requirement depends upon the dimension of solar panel, because cell tower and control room have fixed dimension. This factor is important, because SS-BS is deployed for long run operation. It requires replacement of some life-limited components due to long period of operation, e.g., battery units. Over-dimensioning leads to unnecessary increased investment, whereas under-dimensioning leads to frequent energy outage.

2.1 State-of-the-Art

In cellular systems, user traffic variation is highly random. Therefore, BS dimensioning requires to capture the randomness of harvested energy as well as traffic-dependent energy consumption. Random nature of energy generation by PV panel and dynamic energy flow behavior due to day-night cycle pose major problem while dimensioning the system. Dimensioning studies can be classified into two major categories: numerical methods [6], [8], [9], [10], [11], [12], [13], [14], [15], [16] and analytical methods [17], [18], [19], [20]. Numerical methods are based on system simulation; and they offer better accuracy but involve extensive computation. In [6], the month with minimum energy generation is considered as a critical parameter, which leads to over-dimensioning. In [8], [9], [10], [11], the CapEx is minimized for a given loss of power supply probability but OpEx is not taken into consideration, whereas constant OpEx is considered in [12]. Markov chain based models are presented for estimation of energy outage in [13], [14], [15], [16]. Daily solar irradiance data is used for constructing Markov chain in [13], [14], which leads to inaccurate dimensioning. Also, these works do not consider OpEx while dimensioning. Hourly solar irradiance data is suggested as a reasonable choice for dimensioning in [21]. The energy harvested, load, and battery levels are modeled using discrete-time Markov chain with hourly solar data profile in [15]. Markov chain based model for energy outage and energy excess probability estimation is presented in [16], where the excess energy that can not be stored is sold to the power grid to earn revenue. CapEx and OpEx both are taken into consideration for obtaining the cost-optimal system dimension in [15], [16]. Days of autonomy criteria, defined as the consecutive number of days over which the energy generated is less than a predefined threshold, is used for dimensioning the storage capacity for a given panel size in [22], [23]. This is not a cost-optimal system; instead it leads to over-dimensioning.

Markov model [24] and Artificial Intelligence [25] based methods are also used to generate solar radiance data for places having no dataset or where large dataset is unavailable. In analytical methods, the random solar data and load data are assumed to follow some well-known distributions, such as Beta distribution [17], exponential distribution [26], and Poisson distribution [27]. Regression based techniques are also reported for dimensioning in [18], [19], [20]. The study in [18] did not consider the battery dynamics, which is a strong assumption, because battery dynamics influences the system performance. The authors in [19], [20] did not consider the randomness of data, which may lead to inaccurate dimensioning. Moreover, the cost-optimality aspect is not discussed in these studies.

To sum up, although a few methods have been reported in the literature on dimensioning of SS-BS, the procedure of estimation of cost-optimal system dimension and the corresponding computational overhead have not been investigated yet. To this end, we also note that the daily solar data used for dimensioning incurs less computation but its accuracy is questionable, whereas hourly data provides better accuracy, but the computational cost incurred is very high.

2.2 Motivation and Contributions

As discussed above, SS-BS dimensioning approaches were studied [13], [15], [16]. However, the considered power model for SS-BS has been the same as that for the BSs with hybrid power supply. We argue that, there is a scope of energy as well as revenue saving in SS-BS system dimensioning, because AC-to-DC conversion is not required in SS-BS system. Moreover till now, cost-optimal system dimension is estimated via the following formulation,

\[
\begin{align*}
\text{minimize} & \quad \text{COST} \\
\text{s. t.:} & \quad \text{OUTAGE} \leq \text{outage threshold}
\end{align*}
\]

Energy outage probability for given system parameters, i.e., panel size and battery, was studied in [13], [14], [15], [16], but the methodology to select cost-optimal system dimension has not been discussed. In the existing practice, exhaustive search is used to estimate the optimal system dimension while satisfying a given energy outage constraint, which is computationally extensive. The existing deployments only consider CapEx optimal system [13], [14]. In contrast, for SS-BS, jointly CapEx, OpEx, and ImpEx optimal system is preferred [15], [16]. However, as compared to only CapEx-optimal dimensioning, computation issue is more severe in case of CapEx, OpEx, and ImpEx optimal dimensioning. The tradeoff for system dimensioning process lies in the accuracy, volume of data used, and computation time. For accurate dimensioning, volume of data with fine granularity is used, which involves high computational time. Accuracy of dimensioning has significant importance in terms of energy outage performance and revenue, whereas the computational overhead translates to processing-related energy consumption. Therefore, a
computationally-efficient framework on optimal dimensioning of the SS-BS is of practical interest.

Key contributions of this work are as follows:

- A power consumption model is presented for SS-BS that relies only on solar energy; the model aids in achieving cost-effective green communication system design. The cost saving up to 12% has been observed by using this model.
- Lower bounds on system dimension, namely, panel size and the number of batteries, are obtained using Gaussian mixture model (GMM).
- By curve fitting technique, battery life is modeled as a function of panel size and number of batteries. The total cost, comprising of CapEx, OpEx, and ImpEx, is proved to be quasiconvex, whereas the energy outage is proved to be a convex function of system dimension. Accordingly, the cost-optimal dimensioning problem is transformed into a convex optimization framework, where the objective function (total cost) is quasiconvex and the constraints (energy outage and lower bound on system dimension) are convex. Convexity ensures the global optimal solution.
- A **Computationally-efficient Energy outage aware Cost-optimal Dimensioning Algorithm (CECoDa)** is presented for system dimensioning for a given energy outage constraint. CECoDa is initialized with obtained lower bound.
- We have investigated the performance of proposed analytical framework with data from several cities of India (New Delhi, Itanagar, Ahmedabad, Kolkata, Jaipur, Mumbai, Hyderabad) and USA (Las Vegas, Kansas, Denver, Texas, Missouri, Illinois, Indiana, New York) situated at diverse geographical locations with different traffic profiles. For brevity of presentation, four cities New Delhi, Itanagar, Las Vegas, and Kansas are considered for illustrative numerical performance results. A comparative study of dimensioning performance of the proposed optimization framework with respect to the exhaustive search method, which is used in existing practice, demonstrates significant reduction in computation overhead with accurate system dimension estimation. Reduction in computation time up to 94% has been noticed.

### 2.4 Paper Organization

The paper is organized as follows. Section 3 presents a new power consumption model for SS-BS and lower bounds on system dimension. The cost function is modeled in Section 4, and the energy outage probability is analyzed in Section 5. The proposed CECoDa is presented in Section 6. The results and discussion in Section 7. The paper is concluded in Section 8.

#### Table 1: List of important variables along with their descriptions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{PV}$</td>
<td>Size of PV panel</td>
</tr>
<tr>
<td>$N_{B}$</td>
<td>Number of battery</td>
</tr>
<tr>
<td>$e_{PV}^{1KW}$</td>
<td>Energy harvested by PV panel size of 1 KW</td>
</tr>
<tr>
<td>$n_c$</td>
<td>Charging efficiency of battery</td>
</tr>
<tr>
<td>$n_d$</td>
<td>Discharging efficiency of battery</td>
</tr>
<tr>
<td>$n_{PVLB}$</td>
<td>Low bound of PV panel</td>
</tr>
<tr>
<td>$N_{BLB}$</td>
<td>Lower bound on number of battery</td>
</tr>
<tr>
<td>$C_{CapEx}$</td>
<td>Capital expenditure</td>
</tr>
<tr>
<td>$C_{OpEx}$</td>
<td>Operational expenditure</td>
</tr>
<tr>
<td>$C_{ImpEx}$</td>
<td>Implementation expenditure</td>
</tr>
<tr>
<td>$C_{Ex}$</td>
<td>Total expenditure</td>
</tr>
<tr>
<td>$C_{PV}$</td>
<td>Cost of reference PV panel size of 1 KW</td>
</tr>
<tr>
<td>$C_{B}$</td>
<td>Cost of a battery</td>
</tr>
<tr>
<td>$A_{PV}$</td>
<td>Area of reference PV panel size of 1 KW</td>
</tr>
<tr>
<td>$Y$</td>
<td>Duration of operation of BS</td>
</tr>
<tr>
<td>$C_{R}$</td>
<td>Rental cost of unit area land</td>
</tr>
<tr>
<td>$DoD$</td>
<td>Depth-of-discharge of battery</td>
</tr>
<tr>
<td>$P_{out}$</td>
<td>Energy outage probability</td>
</tr>
<tr>
<td>$L$</td>
<td>Lifetime of battery</td>
</tr>
</tbody>
</table>

### 3 Evaluation of Lower Bound on System Dimension

SS-BS system model shown in Fig. 1 consists of PV panels as energy harvester, a set of battery as energy storage device, and BS as energy consumer or load. Energy can be extracted from battery whenever the harvested energy by the PV panel is less than the required consumption. If instead, the energy harvested is more than the required consumption, energy is added to the battery. Solar energy profile and power consumption profile of BS are required to be analyzed for cost-effective system dimensioning.

Solar radiation data provides information on how much of the Sun’s energy strikes the solar panel surface during a particular time period. For this purpose, statistical solar data of last 10 years has been accessed from National Renewable Energy Laboratory (NREL). This large volume of data over such a long period also incorporates the seasonal variations. This data is fed into the System Advisor Model (SAM) to get the hourly energy generated by a panel of 1 KW rating i.e., $e_{PV}^{1KW}$ with default settings [29]. SAM is a performance modeling software designed for renewable energy industry. For analysis and illustration, a few geographically diverse locations having different solar energy generation capabilities are considered. Here, two cities from India (New Delhi and Itanagar) and two cities from USA (Las Vegas and Kansas). Itanagar is a hilly area that exhibits significant hourly irradiance variation, whereas New Delhi is a plain land area. Las Vegas has more sunny days than Kansas, whereas Kansas has more rainy days than Las Vegas.
3.1 Proposed Power Consumption Model of SS-BS

The basic power consumption model for BS is given by [30]:

\[ P^{BS}(t) = N_{trx} \cdot (P_o + \Delta \cdot \rho(t) \cdot P_{max}), \]  

(1)

where \( N_{trx} \) is the number of transceivers, \( P_o \) is the power consumption at zero RF output power, \( \Delta \) is the slope of load dependent power consumption, \( P_{max} \) is the maximum RF output power at maximum load, and \( 0 \leq \rho(t) \leq 1 \) is the normalized traffic at time \( t \). The values of these parameters are different for different types of BSs. For macro BS, which is considered for analysis in this work, \( N_{trx} = 6, P_o = 130 \text{ W}, \Delta = 4.7, P_{max} = 20 \text{ W} \), and the hourly normalized traffic profile \( \rho(t) \) is shown in Fig. 2.

The traffic profile considered has been generated from the data collected from LTE macro BS of Vodafone mobile operator with coverage radius of 500 m [31].

The model given in Fig. 1 comprises power consumption by several components, namely power amplifier, radio frequency (RF) transceiver, baseband, DC-DC, cooling, and AC-DC (main supply). Power consumed by each component and the nature of their operating voltage are listed in Table 2 for a typical macro BS [31].

It may be noted that, main supply contributes approximately 8% (18 W), which primarily due to AC to DC power conversion. Then DC-DC converter supplies the required DC voltage to the other components, i.e., power amplifier, RF transceiver, baseband. The cooling system can operate on AC as well as DC supply. Table 2 indicates that the DC power supply is sufficient to operate the SS-BSs, as they are not connected to grid; PV panels generate the required DC power. Therefore, by removing the main supply unit and operating only by the DC power generated from the PV panel can save significant amount of energy in normal as well as sleep mode of operation. This mechanism can also save significant amount of energy in other types of SS-BSs, such as micro, pico and femto BS, where the main supply consumes approximately 8%, 11%, and 11%, respectively [33]. Considering the above modification by removing the main supply unit, the new power consumption model for SS-BS suggested here is given as,

\[ P^{SS-BS}(t) = N_{trx} \cdot (P_o + \Delta \cdot \rho(t) \cdot P_{max}). \]  

(2)

Parameters of the new power consumption model are listed in Table 3.

![Figure 1: System model for stand-alone solar-enabled base station (SS-BS).](image)

![Figure 2: Variation of hourly traffic profile during weekdays and weekends.](image)

3.2 Evaluation of Lower Bound on System Dimension

Using the hourly solar data and power consumption profile (cf. (2)), the lower bound on system dimension (PV panel and storage) is estimated. Let the charging and discharging efficiency of the lead acid battery denote by \( \eta_c \) and \( \eta_d \), respectively. Assume that the system has sufficient storage, such that the energy is available in the battery whenever required. This is a valid assumption while evaluating the lower bounds. Let \( e_{cons} \) denotes the hourly energy consumption by BS obtained from (cf. (2)) using the hourly traffic profile shown in Fig. 2.

### 3.2.1 Modeling of Leftover Energy

Energy harvested by the PV panel of size \( n_{PV} \) (i.e., having \( n_{PV} \) units of solar cells in the panel) is given as,

\[ e_{H}^{npv}(t) = n_{PV} \cdot e_{H}^{1KW}(t), \]  

(3)

where \( t \) denotes the hour index, as hourly data is available.

The battery energy dynamics, which involves charging-discharging phenomena, is given by,

\[ B(t) = B(t-1) + e_{L}^{npv}(t), \]  

(4)

where \( e_{L}^{npv}(t) \) is the leftover energy at time instant \( t \) and given by,

\[ e_{L}^{npv}(t) = \begin{cases} \eta_c (e_{H}^{npv}(t) - e_{cons}(t)), & e_{H}^{npv}(t) \geq e_{cons}(t) \\ 1 - \eta_d (e_{H}^{npv}(t) - e_{cons}(t)), & e_{H}^{npv}(t) < e_{cons}(t) \end{cases}. \]  

(5)

Positive leftover energy indicates that, the harvested energy is greater than energy usage by the load, and the excess energy is added to the battery. Negative leftover energy indicates that, the harvested energy is less than the energy demand by the load, and the required energy is withdrawn from the battery.

Empirically obtained probability distribution function (PDF) of leftover energy at a SS-BS in Las Vegas city is shown in Fig. 3(a). The variation of pdf of leftover energy does not follow any standard distribution; it has several peaks. Therefore, we model...
it by Gaussian mixture model (GMM). GMM is a probabilistic model that assumes all the data points are generated from a mixture of a finite number of Gaussian distributions. Each Gaussian component is associated with a normalized weight, which indicates its proportion in the distribution $[34]$. The hourly data for all the years solar energy generation is considered to parameterize the leftover energy. Large volume of dataset offers better fitting accuracy; it also incorporates the seasonal variations.

Let $E_{nPV}^h$ denote the random variable that captures the variation of hourly leftover energy $E_{nPV}^h$ for a given $n_{PV}$. The GMM modeled distribution of $E_{nPV}^h$ in [5], i.e., the distribution of the random variable $E_{nPV}^h$, is expressed as:

$$f_{E_{nPV}^h}(e) = \sum_{i=1}^{K} \frac{\alpha_i^{n_{PV}}}{\sqrt{2\pi} \cdot \sigma_i^{n_{PV}}} \cdot \exp \left( -\frac{(e - \mu_i^{n_{PV}})^2}{2 \cdot (\sigma_i^{n_{PV}})^2} \right),$$

where $K$ is the number of Gaussian components used to model the data, $\alpha_i^{n_{PV}}$, $\mu_i^{n_{PV}}$, and $\sigma_i^{n_{PV}}$ are respectively weight, mean, and standard deviation of the $i^{th}$ Gaussian component for a given panel size $n_{PV}$. The weights of all components in GMM satisfies $\sum_{i=1}^{K} \alpha_i^{n_{PV}} = 1$.

In Fig. [3](a), the negative index of $E_{nPV}^h$ denotes the energy extracted from the storage, whereas positive index denotes the energy added to the storage. Kolmogorov-Smirnov (KS) test is used to identify the suitable number of Gaussian components ($K$) for fitting [35]. KS distance is the maximum distance between empirical cumulative distribution function (CDF) and estimated CDF using GMM. When the change in KS distance between two successive GM models is below a threshold, then addition of number of components is stopped. The CDF of empirical data and GMM are shown in Fig. [3](b) for different panel size at a SS-BS in Las Vegas city. For a given KS distance threshold 0.05, the best GMM fit for the four cities, New Delhi, Itanagar, Las Vegas, and Kansas, with $n_{PV} = 5$ KW are achieved with $K = 5, 7, 6,$ and 6, respectively.

### 3.2.2 Lower Bound on System Dimension

Using the developed GMM distribution of leftover energy in (6), the mean of hourly leftover energy is obtained as,

$$E\left[ E_{nPV}^h \right] = \int_{-\infty}^{\infty} e \cdot f_{E_{nPV}^h}(e) \cdot de = \sum_{i=1}^{K} \alpha_i^{n_{PV}} \cdot \mu_i^{n_{PV}}.$$ (7)

For a sustainable system in the long run, the mean of hourly leftover energy should be positive to avoid the energy outage in each hour on an average. This indirectly indicates the energy neutrality on hourly basis, and overall, the energy is added to storage in each hour. If instead the mean leftover energy is negative, then on average the energy is extracted from storage in each hour, thereby leading to depletion of stored energy and subsequent energy outage.

Thus, for a green sustainable system in long run, $\mathbb{E} \left[ E_{nPV}^h \right] \geq 0$. Accordingly, the lower bound on System dimension $n_{PV, LB}$ is expressed as:

$$n_{PV, LB} = \inf \left\{ n_{PV} \mid \sum_{i=1}^{K} \alpha_i^{n_{PV}} \cdot \mu_i^{n_{PV}} \geq 0 \right\},$$ (8)

where $\inf$ indicates the infimum.

The mean energy extracted from storage in each hour $S_{n_{PV}}^h$ for a given panel size $n_{PV}$ corresponds to the negative index of leftover energy profile and is expressed as:

$$S_{n_{PV}}^h = \int_{-\infty}^{0} e \cdot f_{E_{nPV}^h}(e) \cdot de$$

$$= \int_{-\infty}^{0} e \cdot \frac{\alpha_i^{n_{PV}}}{\sqrt{2\pi} \cdot \sigma_i^{n_{PV}}} \cdot \exp \left( -\frac{(e - \mu_i^{n_{PV}})^2}{2 \cdot (\sigma_i^{n_{PV}})^2} \right) \cdot de,$$

$$= \sum_{i=1}^{K} \frac{\alpha_i^{n_{PV}}}{\sqrt{2\pi} \cdot \sigma_i^{n_{PV}}} \cdot \left[ \sqrt{2\pi} \cdot \mu_i^{n_{PV}} \cdot \left( 1 - Q\left( \frac{-\mu_i^{n_{PV}}}{\sigma_i^{n_{PV}}} \right) \right) - \sigma_i^{n_{PV}} \cdot \exp \left( -\frac{(\mu_i^{n_{PV}})^2}{2 \cdot (\sigma_i^{n_{PV}})^2} \right) \right],$$

(9)

where $Q(\cdot)$ denotes the Q-function.

Typically, solar radiation is available during day hours only. The 24 hours period constitutes a cycle over which charging and discharging occurs. On average, the total energy extracted in each cycle is $S_d^{n_{PV}} = 24 \cdot S_{n_{PV}}^h$.

Therefore, for a panel size $n_{PV}$, the storage capacity required $S_{n_{PV}}$ is governed by,

$$S_{n_{PV}} \geq |S_d^{n_{PV}}|$$

$$= 24 \cdot \left\lfloor \sum_{i=1}^{K} \frac{\alpha_i^{n_{PV}}}{\sqrt{2\pi} \cdot \sigma_i^{n_{PV}}} \cdot \left[ \sigma_i^{n_{PV}} \cdot \exp \left( -\frac{(\mu_i^{n_{PV}})^2}{2 \cdot (\sigma_i^{n_{PV}})^2} \right) \right] \right\rfloor,$$

(10)

where $S_{n_{PV}}^{LB}$ is the lower bound on storage. It indicates that, at least this amount of energy should be available, such that whenever harvested energy is less than the required consumption, the storage is able to supply. $S_{n_{PV}}^{LB}$ also depends on the panel size. Thus, the lower bound on number of battery units can be obtained as,

$$N_{LB} = \left[ \frac{S_{n_{PV}}^{LB}}{\delta \cdot \delta_{cap}} \right],$$

(11)

where $\delta_{cap}$ is the capacity of each battery and $\delta$ is the depth-of-discharge of each battery.

**Remark 1.** For very large panel size, the storage required during radiation hours reduces to very small or insignificant value, whereas storage requirement remains vital during absence of radiation. Let us denote energy storage requirement in absence of radiation by $N_{Bh}$; it can be obtained as,

$$S_{th} = \lim_{n_{PV} \to \infty} S_{n_{PV}}^{LB} \Rightarrow N_{Bh} = \left[ \frac{S_{th}}{\delta \cdot \delta_{cap}} \right].$$

(12)

It is notable that, unlike $N_{LB} \cdot N_{Bh}$, independent of panel size with $N_{LB} \geq N_{Bh}$, and due to its independency from $n_{PV}$,
it is used in the formulation of optimization problem in subsequent sections.

4 Modeling of Cost Function

In green sustainable system, it is important to choose the number of PV panels and battery units in cost-optimal sense, such that the long-run energy outage remains within an acceptable range. The expenditure in the given context refers to the cost of PV panel and battery, which comprises of mainly three parts: CapEx, OpEx, and ImpEx. Other costs, such as, components, manpower, and transportation are supposed to be constant and have no impact on system dimensioning.

4.1 CapEx

CapEx accounts for the initial investment, which includes the dimensioning-related costs, namely, the cost of PV panel, battery, and equipment. Accordingly, CapEx of SS-BS is captured as:

\[ C_{\text{CapEx}}(n_{PV}, N_B) = C_{PV} \cdot n_{PV} + C_B \cdot N_B + C_0, \]  

(13)

where \( C_{PV} \) and \( C_B \) denote the cost of reference panel of size 1 KW and cost of a battery unit, respectively. \( C_0 \) captures the constant costs.

\( C_0 \) includes the cost of BS components, such as digital signal processor, power amplifier, transceiver, antenna, cellular tower, etc. The cost of these components do not depend on \( n_{PV} \) and \( N_B \). Also, they have sufficiently long lifetime and hence these are deployed once, unlike the battery units which have limited lifetime. Thus, \( C_0 \) is not influenced by the system dimension and is treated as constant in the given context. Therefore, for compactness the actual cost of these components are not considered in the optimization framework.

4.2 OpEx

Usually, SS-BSs are installed for long period of operation, i.e., for 10 to 20 years. Hence, it involves replacement of life-limited components, i.e., battery in the current context. OpEx accounts for this cost, which is explored as:

\[ C_{\text{OpEx}}(n_{PV}, N_B) = C_B \cdot N_B \cdot \frac{Y}{L(n_{PV}, N_B)} - C_B \cdot N_B + C_1, \]  

(14)

where \( Y \) denotes the period of operation in years and \( L(n_{PV}, N_B) \) denotes the lifetime of battery with system dimension \( n_{PV}, N_B \). \( C_1 \) is a constant, which includes other costs. \( C_B \cdot N_B \) term is deducted from OpEx, because it is included in CapEx at the time of installation.

Battery life is an important factor in renewable energy based systems, because it has a strong impact on OpEx. In general, lifetime depends on the number of cycles and depth-of-discharge (DoD) in each cycle. DoD indicates the percentage of charge extracted from the battery. A lower DoD per cycle offers a higher lifetime, and vice-versa. Due to random nature of energy harvested and absence of solar irradiance during night hours, the DoD varies significantly over each cycle, which causes degradation of battery life. Rainflow counting algorithm [36] is a widely used technique to count the total number of cycles for a given battery energy dynamics. The number of cycles against DoD data given in the data-sheet of battery [37] is used here to obtain the lifetime of a set of battery units. Using curve fitting technique, the number of cycles against the DoD is found to vary as:

\[ N_c(DoD) = 7855 \cdot \exp(-9.48 \cdot DoD) + 2508 \cdot \exp(-1.605 \cdot DoD). \]  

(15)

The variation of fitted and original number of cycles against the DoD is shown in Fig. 4 which depicts excellent fitting with R-squared value 0.9994.

The temperature dependence of number of cycles is captured as:

\[ N_c(DoD, T) = N_c(DoD) \cdot (37.68 \cdot T^{-1.101} - 0.3897), \]  

(16)

where \( T \) denotes the temperature in °C. Here, \( T = 27^\circ \text{C} \) is considered, where full battery capacity is available [37] and this temperature can be maintained using cooling system.

The battery life for a given system dimension \( n_{PV}, N_B \) is evaluated as:

\[ L(n_{PV}, N_B) = \left( \sum_{k=1}^{N_c} \frac{1}{N_c(DoD_k, T=27^\circ \text{C})} \right)^{-1}, \]  

(17)

where \( N_c \) is the total number of cycles obtained from Rainflow counting algorithm [36] and \( N_c(DoD_k) \) is the number of cycles corresponding to DoD of \( k^{th} \) cycle. Battery life depends on the panel size as well as the number of battery units, because the energy harvested by panel dictates the DoD of battery.

4.3 ImpEx

ImpEx accounts for the cost to rent a site, construction, and installation. The rental cost depends on the area of site required to set up the BS as well as to install PV panel. Thus, the ImpEx of SS-BS is expressed as,

\[ C_{\text{ImpEx}}(n_{PV}, N_B) = A_{PV} \cdot n_{PV} \cdot Y \cdot C_R + C_2, \]  

(18)

where \( A_{PV} \) denotes the area covered by a reference panel of size 1 KW rating, \( C_R \) denotes the rent per unit area per year, \( Y \) denotes the number of years for operation. \( C_2 \) is a constant, which includes the other costs, namely, construction and installation costs. The area covered by BS set up is incorporated in constant part and only PV panel size dependent rent is taken into consideration.
4.4 Characterization of Cost Function

The total expenditure, i.e., cost function is given as,

\[
C_{Ex}(n_{PV}, N_B) = C_{CapEx}(n_{PV}, N_B) + C_{OpEx}(n_{PV}, N_B) + C_{ImpEx}(n_{PV}, N_B)
\]

\[
= C_{PV} \cdot n_{PV} + C_B \cdot \frac{Y}{L(n_{PV}, N_B)} \cdot N_B + C_R \cdot n_{PV} \cdot A_{PV} \cdot Y + C_{all},
\]

(19)

where \( C_{all} = C_0 + C_1 + C_2 \) accounts for all constants.

The expression of battery life in (17) is not analytically tractable, because it depends upon the depth-of-discharge of each cycle and the number of cycles. Both of these quantities have no closed form expression as function of \( n_{PV} \) and \( N_B \), because the depth-of-discharge as well as number of cycle is dictated by the battery dynamics. Therefore, the battery life for each system combination \((n_{PV}, N_B)\) is calculated, and its nature is explored using an empirical equation obtained from curve fitting technique [38]. In order to examine the accuracy of fitting, two goodness-of-fit parameters R-square and root mean square error (RMSE) have been used.

Battery life for different system dimension \((n_{PV}, N_B)\), where each battery has capacity rating 12 V, 205 Ah with \( \delta = 0.7 \), is obtained and fitted using curve fitting technique. The following empirical equation captures the battery life as function of \( n_{PV} \) and \( N_B \).

\[
L(n_{PV}, N_B) = a_{00} + a_{10} \cdot n_{PV} + a_{01} \cdot N_B + a_{11} \cdot n_{PV} \cdot N_B + a_{20} \cdot n_{PV}^2 + a_{02} \cdot N_B^2 + a_{21} \cdot n_{PV} \cdot N_B^2 + a_{12} \cdot n_{PV} \cdot N_B^2 + a_{30} \cdot n_{PV}^3 + a_{03} \cdot N_B^3.
\]

(20)

The fitting coefficients \( a_{ij} \)'s for four different cities are listed in Table 4 along with the values of goodness-of-fit parameters. The R-square value close to 1 and very small RMSE value indicate the accuracy of curve fitting.

**Remark 2.** We restrict our analysis for system dimension in the range: \( n_{PV, LB} \leq n_{PV} \leq 20 \text{ KW}, N_{B,ab} \leq N_B \leq 75 \), which is sufficient for practical deployment perspective of SS-BS. Consideration of a higher system dimension does not have practical consequence, although the presented analysis remains valid for higher system dimension as well.

**Remark 3.** The panel size \( n_{PV} \) and the number of battery units \( N_B \) are considered as continuous variables for characterization of the cost-optimal dimensioning problem. Actually, storage and panel size are continuous variables; but in practical deployment scenario, the number of battery units is integer-valued and some fixed increment in panel size, e.g., 0.5 KW or 1 KW is considered. This assumption is used only to characterize the cost-optimal dimensioning problem; it will not affect the cost-optimal solution.

**Theorem 1.** The total expenditure \( C_{Ex}(n_{PV}, N_B) \) is quasiconvex function of system dimension, i.e. panel size \( n_{PV} \) and the number of battery units \( N_B \).

**Proof:** See Appendix A

Theorem 1 can be interpreted as follows: When the deployed number of PV cells \( n_{PV} \) and battery units \( N_B \) are small, the CapEx is less. However, this small value of \( n_{PV} \) and \( N_B \) lead to frequent discharge of the battery units due to lesser energy generation and energy storage capacity. Consequently, this causes a higher DoD in charging cycles which results in effectively less number of tolerable charging cycles of battery units, and hence its lifetime is reduced. Thus, more frequent requirement of battery replacement causes a higher OpEx. On the other hand, CapEx is very high when the deployed numbers of \( n_{PV} \) and \( N_B \) are high. However, the battery does not discharge severely, i.e., DoD is lesser due to high energy generation as well as energy storage capacity. A lesser DoD in a charging cycle in turn offers effectively a high number of charging cycles. Hence, the lifetime of a battery unit is longer, which leads to a lesser OpEx. These extreme cases exist at boundary; the cost decreases in between them, thus offering a quasiconvex nature of the total expenditure.

### Table 4: Fitting coefficients and the values of goodness-of-fit parameters for battery life, for different cities

<table>
<thead>
<tr>
<th>City</th>
<th>( a_{00} )</th>
<th>( a_{10} )</th>
<th>( a_{01} )</th>
<th>( a_{11} )</th>
<th>( a_{20} )</th>
<th>( a_{02} )</th>
<th>( a_{21} )</th>
<th>( a_{12} )</th>
<th>( a_{30} )</th>
<th>( a_{03} )</th>
<th>( R^2 )</th>
<th>( \text{RMSE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Delhi</td>
<td>(-20.77)</td>
<td>5.421</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.7538</td>
<td>0.0798</td>
</tr>
<tr>
<td>Itanagar</td>
<td>(-23.25)</td>
<td>10.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5387</td>
<td>0.0563</td>
</tr>
<tr>
<td>Las Vegas</td>
<td>(-40.74)</td>
<td>15.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.7476</td>
<td>0.0600</td>
</tr>
<tr>
<td>Kansas</td>
<td>(-4.273)</td>
<td>1.525</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8198</td>
<td>0.0398</td>
</tr>
</tbody>
</table>

5 Analysis of Energy Outage Probability

Energy outage happens when neither PV panel nor storage is able to supply the required energy. Energy availability, which is the complement of energy outage, is measured using number of 9’s [39]. For example, two 9’s indicate 99% availability, or 1% energy outage, which means about 15 minutes of energy blackout daily. Up to 1% energy outage is considered acceptable in cellular BS operation [4]. Maintaining a low energy outage is also important because a BS takes several minutes to start functioning after a shut-down [40]. The energy outage probability for a given panel size and the number of battery is calculated from (21) using the hourly harvested energy. Here \( B(t) \) is the energy level of battery at time \( t \), \( N_B \) is the number of battery, \( B_{cap} \) is the capacity of each battery unit, \( \epsilon_{PV}^t(t) \) is the hourly energy harvested by panel of size \( n_{PV} \), \( \epsilon_{con}(t) \) is the hourly energy consumed by BS at time \( t \), and \( \delta \) is the depth-of-discharge of battery. \( I(\cdot) \) is an indicator function, given as:

\[
I(x) = \begin{cases} 
  x, & \text{if } x \geq 0 \\
  0, & \text{if } x < 0.
\end{cases}
\]
outage probability as a function of system dimension is captured and its variation as a function of $n$ for analyzing the system dimensioning further. For this purpose, curve fitting technique is used to obtain an empirical expression $n$ its DoD. The operation is not allowed beyond this range.

The battery dynamics restrict the analytical tractability of variation of energy outage probability as a function of system dimension, i.e., PV panel size. Theorem 2.

Table 5: Fitting coefficients and the values of goodness-of-fit parameters for energy outage probability, for different cities

<table>
<thead>
<tr>
<th>City</th>
<th>$b_{00}$</th>
<th>$b_{10}$</th>
<th>$b_{01}$</th>
<th>$b_{02}$</th>
<th>$b_{11}$</th>
<th>$b_{12}$</th>
<th>$B_{min}$</th>
<th>$B_{opt}$</th>
<th>$B_{max}$</th>
<th>$R$-square</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Delhi</td>
<td>0.01413</td>
<td>20.28</td>
<td>-0.0006561</td>
<td>7372</td>
<td>-0.2659</td>
<td>6.71 $\times$ 10^{-6}</td>
<td>0.0043</td>
<td>0.9434</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Itanagar</td>
<td>0.01413</td>
<td>121.83</td>
<td>-0.001797</td>
<td>11391</td>
<td>-0.1121</td>
<td>1.691 $\times$ 10^{-5}</td>
<td>0.00109</td>
<td>0.9664</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Las Vegas</td>
<td>0.01888</td>
<td>58.80</td>
<td>-0.0008892</td>
<td>2886</td>
<td>-0.2907</td>
<td>9.023 $\times$ 10^{-6}</td>
<td>0.0047</td>
<td>0.9667</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kansas</td>
<td>0.01729</td>
<td>17.21</td>
<td>-0.0006997</td>
<td>137.8</td>
<td>-0.1327</td>
<td>6.582 $\times$ 10^{-6}</td>
<td>0.0027</td>
<td>0.9384</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Fitting coefficients and the values of goodness-of-fit parameters for energy outage probability, for different cities

Here also, highly random nature of harvested energy and battery dynamics restrict the analytical tractability of variation of outage probability as a function of system dimension, i.e. $n_{PV}$ and $N_B$. Equation (21) is an iterative expression, where the batteries can be charged up to its maximum capacity and discharged up to its DoD. The operation is not allowed beyond this range.

A closed-form expression of energy outage probability as a function of $n_{PV}$ and $N_B$ is not available. To this end, in this work curve fitting technique is used to obtain an empirical expression for analyzing the system dimensioning further. For this purpose, energy outage probability for each system dimension is calculated, and its variation as a function of $n_{PV}$ and $N_B$ is obtained. Energy outage probability as a function of system dimension is captured using the following empirical equation:

$$P_{out}(n_{PV}, N_B) = b_{00} + b_{10} \cdot e^{-n_{PV}} + b_{01} \cdot N_B + b_{11} \cdot e^{-n_{PV} \cdot N_B} + b_{20} \cdot e^{-2n_{PV}} + b_{02} \cdot N_B^2.$$  (22)

The fitting coefficients $b_{ij}$’s for four different cities are listed in Table 5 along with the goodness-of-fit values. The $R$-square values are close to 1 and RMSE values are very small, which indicate the accuracy of the empirical expression.

It may be noted from (22) that, the energy outage probability decreases with increase in $n_{PV}$ and $N_B$ values, but the decay is exponential with increase of $n_{PV}$. Larger PV panel generates higher energy, which leads to sufficient energy availability in the battery units, and thus it reduces the outage probability.

**Theorem 2.** The energy outage probability $P_{out}(n_{PV}, N_B)$ is convex function of system dimension, i.e., PV panel size $n_{PV}$ and number of battery units $N_B$.

**Proof:** See Appendix [B]  

**Remark 4.** The cost-optimal dimensioning problem while satisfying the energy outage constraint is convex. Therefore, the local optimal solution overlaps with the global optimal solution.

**Remark 4.** The cost-optimal dimensioning problem while satisfying the energy outage constraint is convex. Therefore, the local optimal solution overlaps with the global optimal solution.

The emphasis is on characterizing the optimization problem (P1) rather than solving it, because it provides the clue about the variation of function, and indicates where to stop the process. In this work, we will take the advantage of convexity of (P1) to reach the optimal solution in computationally efficient way. On the other hand, one needs the empirical equations (20) and (22) in order to solve (P1) for a given city. This requires computation of battery life (cf. (7)) and outage probability (cf. (21)) by exhaustive search, involving significant overhead. Also, the cost-optimal system dimension should be integer valued from real life deployment perspective, but (P1) does not provide integer solution. Rounding the non-integer value to integer value will lead to over-dimensioning or under-dimensioning. Thus, solving (P1) is neither computationally efficient nor serves the purpose. On the other hand, the empirical equations (20) and (22) are obtained to characterize the system behavior, which remains valid in general.

Although convexity ensures the global optimal solution, our aim is to reach this cost-optimal solution $(n_{PV, opt}, N_{B, opt})$ in minimum time. Therefore, we need to come up with a procedure

$$B(t) = \begin{cases} \frac{N_B \cdot B_{cap}}{B(t-1) + n_c \cdot I(e_{PV}^{n_{PV}}(t) - e_{cons}(t)) + \frac{1}{\eta_d} \cdot I(e_{cons}(t) - e_{PV}^{n_{PV}}(t))}, & \text{if } B(t) \geq N_B \cdot B_{cap} \\ (1 - \delta) \cdot N_B \cdot B_{cap}, & \text{if } (1 - \delta) \cdot N_B \cdot B_{cap} < B(t) < N_B \cdot B_{cap} \\ 1 \cdot B_{min}, & \text{if } B(t) \leq (1 - \delta) \cdot N_B \cdot B_{cap}. \end{cases}$$

6 Computationally-efficient Cost-Optimal Dimensioning

Till now, the feasible search space is identified in Section 3 and cost function and energy outage are characterized in Sections 4 and 5, respectively. In this section, the cost-optimal system dimensioning is performed using the analysis in previous sections. The optimization problem for cost-optimal dimensioning while satisfying the energy outage constraint is formulated as:

$$\begin{align*} \text{(P1)} : \min_{(n_{PV, N_B})} & \quad C_{Ex}(n_{PV}, N_B) \\
\text{subject to:} & \quad P_{out}(n_{PV}, N_B) \leq p, \\
& \quad -n_{PV} \leq -n_{PV, LB}, \\
& \quad -N_B \leq -N_{B,ub}, \\
\text{where} & \quad C_{Ex}(n_{PV}, N_B) = C_{CapEx}(n_{PV}, N_B) + C_{OpEx}(n_{PV}, N_B) + C_{ImpEx}(n_{PV}, N_B) = C_{PV} \cdot n_{PV} + C_{B} \cdot \frac{N_B + C_{R} \cdot N_B + A_{PV} \cdot Y + C_{all}}{E(n_{PV}, N_B)}.
\end{align*}$$

Theorem 1 and Theorem 2 transform the optimization problem (P1) into a convex optimization framework [41], which ensures a global optimal.

Theorem 2. The energy outage probability $P_{out}(n_{PV}, N_B)$ is convex function of system dimension, i.e., PV panel size $n_{PV}$ and number of battery units $N_B$.

**Proof:** See Appendix [B]  

Theorem 2 can be interpreted as follows: For smaller $n_{PV}$ and $N_B$, deployment, frequent energy outage happens due to lesser energy generation and energy storage capacity. On the other hand, larger $n_{PV}$ and $N_B$ avoid energy outage due to higher energy generation and energy storage capacity. Also, with increase in either $n_{PV}$ or $N_B$, the harvested energy or storage capacity increases, which reduces the energy outage. This indicates that, the energy outage will improve significantly with increase in system dimension, which leads to the convex nature.
such that the minimum number of system combinations could be traversed while achieving an optimal solution. The path to optimal solution can be traversed in depth first search fashion due to the discrete nature of variables \((n_{PV} \text{ and } N_B)\), where the search for optimal battery dimension can be performed for a given panel size. This restricts the search process in one-dimensional space rather than in two-dimension; only one variable changes, the other remains constant. Therefore, for a given panel size \(n_{PV} = n_{PV,0}\), the optimal number of battery units can be obtained from the following optimization problem:

\[(P2) : \text{minimize} \quad C_B \cdot \frac{Y}{L(n_{PV,0}, N_B)} \cdot N_B + C_{PV} \cdot n_{PV,0} + A_{PV} \cdot n_{PV,0} \cdot Y \cdot C_R + C_{all} \]

subject to: \(P_{out}(n_{PV,0}, N_B) \leq p\),
\[-N_B \leq -N_{B_{\text{thr}}}.

It is important to characterize the optimization problem \((P2)\) in order to find the optimal solution efficiently, as the nature of optimization problem will provide the clue to reach the optimal solution in a lesser time.

Lemma 1. The total expenditure \(C_{Ex}(n_{PV}, N_B)\) is a quasiconvex function of number of battery units \(N_B\) for a given panel size.

Proof: See Appendix C.

Lemma 2. The outage probability \(P_{out}(n_{PV}, N_B)\) is a convex function of number of battery units \(N_B\) for a given panel size.

Proof: See Appendix D.

Lemma 1 and Lemma 2 assure that \((P2)\) is also a convex optimization problem, as the objective function is quasiconvex function of \(N_B\) and the constraints are convex function of \(N_B\). Hence the optimal number of battery obtained from \((P2)\) is global optimal for a given panel size, and there is no need to search for all values of \(N_B\).

Remark 5. For a given panel size, an optimal number of battery units obtained while satisfying the energy outage constraint is also global optimal due to convex nature of the optimization framework.

Computationally-efficient Energy outage aware Cost-optimal Dimensioning Algorithm (CECoDA) is proposed in Algorithm 1 by taking advantage of convexity of the optimization problems \((P1)\) and \((P2)\). If energy outage constraint is not satisfied, then the total expenditure, which includes computation of battery life, is not calculated in order to save time. The optimal selection process starts from the lower bound of panel size \(n_{PV,LB}\). Then the optimal number of battery units is estimated while satisfying the energy outage probability for this given panel size. The search stops when increase in cost function is noticed due to convex nature of \((P2)\) (cf. Remark 5). Then, the panel size is increased by \(\nabla n_{PV}\) and the optimal number of battery units is obtained. The cost of optimal combination with present panel size is compared with that of the previous panel size, and this process is repeated till the decrease in cost is noticed. This procedure yields optimum result due to convex nature of problem \((P1)\) (cf. Remark 4). The search process stops due to convexity of \((P1)\) and \((P2)\), which ensure a global optimal solution. \(\nabla n_{PV}\) is the incremental value of the PV panel size; \(\nabla n_{PV} = 1\) KW is considered in this work.

Algorithm 1  Computationally-efficient Energy outage aware Cost-optimal Dimensioning Algorithm (CECoDA)

1. **Required**: energy harvesting data, traffic profile, \(\eta_c, \eta_d, p, C_{PV}, C_B, C_R, A_{PV}, \nabla n_{PV}\)
2. **Ensured**: \((n_{PV}^{opt}, N_B^{opt})\)
3. start with \(n_{PV} = n_{PV, LB}\)
4. start with \(N_B = N_{B, LB}\)
5. calculate \(P_{out}(n_{PV}, N_B)\) for given \(n_{PV}\) and \(N_B\)
6. **if** \(P_{out}(n_{PV}, N_B) \leq p\) **then**
   7. **compute** \(C_{Ex}(n_{PV}, N_B)\)
   8. **else**
   9. **break**
10. **end if**
11. **if** \(C_{Ex}(n_{PV}, N_B) < C_{Ex}(n_{PV}, N_B - 1)\) **then**
12. \(N_B = N_B + 1\)
13. **else**
14. **break**
15. **end if**
16. \(\mathcal{R}(n_{PV}) = N_B - 1\)
17. **if** \(C_{Ex}(n_{PV}, \mathcal{R}(n_{PV})) < C_{Ex}(n_{PV} - \nabla n_{PV}, \mathcal{R}(n_{PV}) - \nabla n_{PV})\) **then**
18. \(n_{PV} = n_{PV} + \nabla n_{PV}\)
19. **goto** step 4
20. **else**
21. **stop**
22. **end if**
23. Cost optimal system dimension: \(n_{PV}^{opt} = n_{PV} - \nabla n_{PV}\) and \(N_B^{opt} = \mathcal{R}(n_{PV}^{opt})\)
24. **Optimal cost**: \(C_{Ex}^{opt} = C_{Ex}(n_{PV}^{opt}, N_B^{opt})\)

7 RESULTS AND DISCUSSION

Four cities New Delhi, Las Vegas, and Kansas located at different geographical locations as well as having diverse solar potential are considered for presenting validation results of the proposed framework. NREL data for several other cities, namely, Ahmedabad, Itanagar, Kolkata, Jaipur, Texas, Denver, Missouri, Illinois, Indiana, New York, have also been used for extensive testing and validation purpose. For brevity of presentation, the results on these cities are not included here. The values of battery efficiency parameters are \(\eta_c = 0.9\) and \(\eta_d = 0.9\). The power model for macro SS-BS is given in (7). The equation parameters are listed in Table 3 and traffic profile is shown in Fig. 2. Flooded lead acid battery with 12 V, 205 Ah capacity having depth-of-discharge \(\delta = 0.7\) is considered. The cost of 1 KW rating PV panel is \(C_{PV} = $1000\); the cost of one battery is \(C_B = $280\). The time duration for long-run operation is considered as \(Y = 10\) years.

7.1 Validation of Lower Bounds

The lower bound on PV panel size for different cities are obtained as: \(n_{PV, LB} = 6\) KW for New Delhi, 8 KW for Itanagar, 5 KW for Las Vegas, and 7 KW for Kansas. The variation of energy outage probability for four different PV panel sizes in the neighborhood of lower bound, i.e., \(n_{PV} = n_{PV, LB} - 1, n_{PV, LB}, n_{PV, LB} + 1, \) and \(n_{PV, LB} + 2\) against the number of battery units for the four cities are shown in Fig. 3. The panel size \(n_{PV} = n_{PV, LB} - 1\) is not able to harvest sufficient energy required by load and frequent energy outage occurs due to this. The energy outage probability saturates for \(n_{PV} = n_{PV, LB} - 1\) and no improvement in energy outage is observed even for higher number of battery unit.
outage probability is far from acceptable range for validating the bounds, simulation up to 75 batteries (approximately the same value with increase in the number of PV panels, and that, the minimum storage required for all four cities tend towards two parts: one in presence of solar radiation (day hours) and other bound on storage capacity decreases. If the storage is divided in It can be observed that, as the PV panel size increases, the lower number of battery units, \( N_B \) and with \( n_{PV} < n_{PV,LB} \), energy outage is faraway from acceptable range in spite of very large storage deployment.

Remark 6. If the PV panel size is less than its lower bound, i.e. \( n_{PV} < n_{PV,LB} \), energy outage is faraway from acceptable range in spite of very large storage deployment.

Remark 7. If the number of battery units is less than its lower bound, i.e. \( N_B < N_{BLB} \), energy outage is faraway from acceptable range in spite of very large PV panel deployment.

7.2 Accuracy of Cost-optimal System Dimensioning
The cost-optimal system dimension using the proposed CECoDA is obtained for three different energy outage probability constraints: \( p = 1\% \), 0.1\%, and 0.01\%; and the dimension estimates are compared with those offered by three different competitive dimensioning methods: CapEx-OpEx-ImpEx optimal dimensioning [15], [16]. CapEx optimal dimensioning [19], and dimensioning based on days of autonomy criteria [23]. In Table 6, the estimated system dimension with these competitive methods are listed, with \( C_R = 0 \), i.e., the effect of implementation expenditure is not considered. It is noted that, CECoDA estimates the system dimension very accurately, and the optimal system dimension matches closely with that in [15], [16]. The cost of optimal system dimension using the methods reported in [19] and [23] is much higher compared to that offered by CECoDA. This indicates that, system dimensioning based on CapEx-optimality and that with days of autonomy criteria are not good choices for long-run operation of SS-BS. The impact of site rent on optimal system dimension is listed in Table 7 with \( C_R = $100 \) and \( A_{PV} = 5 \text{ m}^2 \). In this scenario also, the approaches in [19] and [23] lead to over-dimensioning of SS-BS. In case of non-zero rent cost, the optimal panel size is relatively lower than that with zero rent cost. It may be observed that, relatively larger system dimension is required to satisfy the stringent energy outage constraint in both the cases, i.e., with \( C_R = $0 \) and with \( C_R = $10 \). The cost-optimal system dimension also indicates that, the solar potential of New Delhi is higher than Itanagar, whereas the solar potential of Las Vegas
### 7.3 Computation Efficiency of CECoDA

The cost-optimal system dimension (cf. Table 6 and Table 7) are listed in Table 8 and Table 9 for $C_R = 0$ and $C_R = 10$, respectively. The simulation is carried out in Matlab 2015 software with Intel Core i7, 64-bit, 3.40 GHz processor having 16 GB random-access memory (RAM). The power consumption of the computer is 240 W. Significant saving in computation time up to 68%-94% by CECoDA is observed, which shows its importance in speeding up the dimensioning process. As the energy outage constraint becomes more stringent, time elapsed in computation increases due to requirement of relatively larger cost-optimal system dimension to satisfy the stringent energy outage constraint. The saving in computation time leads to huge amount of energy saving, which is shown in Fig. 8 Reduction in computation time or energy ranges in 68%-94%.

### 7.4 Cost Saving by using Modified Power Model

The cost saving, which is the difference of total cost of cost-optimal dimension from power model in [1] and the proposed

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### Table 6: Comparison of system dimension and cost estimates obtained by CECoDA with respect to the existing competitive methods, with $C_R = 0$

<table>
<thead>
<tr>
<th>City</th>
<th>$N_{B1}$</th>
<th>$N_{B2}$</th>
<th>$N_{B3}$</th>
<th>$N_{B4}$</th>
<th>$C_{R}$</th>
<th>$C_{P}$</th>
<th>$C_{S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Delhi</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>Itanagar</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>Las Vegas</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>Kansas</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

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Figure 7: Variation of energy outage probability versus PV panel size, with number of battery units $N_{B1} - 1$, $N_{B2}$, $N_{B3} + 1$, and $N_{B4} + 2$ for four different cities.

Figure 8: Saving in computation energy to find the cost-optimal system dimension in the CECoDA over exhaustive search for four different cities having different energy outage constraint.
Table 7: Comparison of system dimension and cost estimates obtained by CECoDA with respect to the existing competitive methods, with $C_R = $10

<table>
<thead>
<tr>
<th></th>
<th>CECoDA [15, 16]</th>
<th>[19]</th>
<th>[23]</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Delhi</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p = 1%</td>
<td>(8.20) $23,760$</td>
<td>(9.21) $24,140$</td>
<td>(11.21) $30,220$</td>
</tr>
<tr>
<td>0.1%</td>
<td>(10.29) $25,360$</td>
<td>(11.22) $26,020$</td>
<td>(13.28) $31,260$</td>
</tr>
<tr>
<td>0.01%</td>
<td>(11.49) $28,820$</td>
<td>(14.43) $29,960$</td>
<td>(16.28) $34,640$</td>
</tr>
<tr>
<td>Itanagar</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>(8.23) $24,040$</td>
<td>(9.18) $24,140$</td>
<td>(10.33) $33,200$</td>
</tr>
<tr>
<td>0.1%</td>
<td>(10.50) $28,720$</td>
<td>(11.43) $28,820$</td>
<td>(11.75) $38,060$</td>
</tr>
<tr>
<td>0.01%</td>
<td>(13.74) $36,020$</td>
<td>(18.73) $37,120$</td>
<td>(18.73) $45,480$</td>
</tr>
<tr>
<td>Las Vegas</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>(11.27) $30,220$</td>
<td>(10.32) $30,400$</td>
<td>(15.35) $43,780$</td>
</tr>
<tr>
<td>0.1%</td>
<td>(13.50) $34,620$</td>
<td>(14.47) $35,280$</td>
<td>(16.62) $47,240$</td>
</tr>
<tr>
<td>0.01%</td>
<td>(14.63) $37,520$</td>
<td>(16.52) $38,640$</td>
<td>(17.70) $48,180$</td>
</tr>
</tbody>
</table>

Figure 9: Cost saving for four different cities having different rent costs and different energy outage constraint.

7.5 Advantage of Proposed Framework

The amount of solar radiation reaching to the earth’s surface varies greatly from location to location because of changing atmospheric conditions. Clouds are the predominant atmospheric condition, which determine the amount of solar radiation arriving to the earth surface. Local geographical features, such as mountains, oceans, and large lakes influence the formation of clouds [42]. Aerosols also lead to significant impairment on the solar radiation due to the scattering and absorption. Due to these reasons, even the places located nearby away may receive very different amount of solar energy. The dependency of solar radiation on several factors mentioned above do not allow to generalize the solar radiation profile. Instead, it is required to analyze the dimensioning procedure separately for each individual locations, which is computationally intensive. Towards this, the proposed framework will be useful, which overcomes the location dependency of solar profile from computation point of view while dimensioning. To investigate the location dependency of solar profile, we have selected two sets of locations from India; they belong to the same solar potential region classified by NREL. NREL has classified the regions of India by considering all the influencing factors mentioned above, and evolved a colored heat map, which indicates the solar potential of different regions [https://www.nrel.gov]. The cities of the first set are Nashik, Pune, and Aurangabad, whereas Kargil, Leh, and Srinagar are the cities of the other set. The cities in each group are located nearby.

The lower bound on PV panel size for cities Nashik, Pune, and Aurangabad are 5 KW, 6 KW, and 6 KW, respectively; whereas the lower bound on PV panel size for cities Kargil, Leh, and Srinagar are 6 KW, 7 KW, and 8 KW, respectively. It is notable that, the lower bound for each set of cities are different, and cannot be generalized based on locality. This optimal system dimension for different energy outage constraint are listed in Table 10 for cities of each set. The cost-optimal system dimension is quite different for the cities of same set in spite of being in the same locality and same color zone of heat map. This observation reveals that, every location has its unique solar profile and needs separate analysis. In order words, the deployment of SS-BS with optimal system dimension obtained for dataset of a particular location will not satisfy the energy outage constraint with the same system dimension for the other nearby locations. This scenario leads to deteriorate the QoE perceived by the user as well as revenue model of operators due to over- or under-dimensioning. To this end, the proposed framework will play a key role by overcoming the location dependency of solar profile is system dimensioning, while sustainability reducing the computation overhead.

8 Conclusion

The power consumption model presented here for solar-enabled system is beneficial for green communication system design, which also saves significant revenue. In this proposed system characterization, the leftover energy has been modeled using Gaussian mixture model. Using this developed distribution, lower bounds on system dimension, i.e., PV panel size and number of battery units,
have been obtained, which determine the feasible search region for dimensioning. In the considered context of SS-BS, out of the total expenditure consisting of CapEx, OpEx, and ImpEx, OpEx depends on battery life, which has been obtained as a function of system dimension using curve fitting technique. The total cost has been proven to be a quasiconvex function, whereas energy outage has been proven to be convex function of system dimension. These properties transform the cost-optimal dimensioning problem into a convex optimization framework, which assures a global optimal solution. A computationally-efficient algorithm called CECoDA has been proposed to estimate the cost-optimal dimension while satisfying the given energy outage constraints without requiring exhaustive search. Four cities with widely varying solar irradiance, namely, New Delhi, Itanagar, Las Vegas, and Kansas, have been considered for numerical evaluation of the proposed dimensioning framework. Substantial saving in computation time and associated processing energy consumption along with excellent accuracy have been observed when applying proposed CECoDA as compared to the competitive exhaustive search based dimensioning. The computationally-efficient framework presented in this work will be very helpful to overcome the location-dependent unique solar profile while dimensioning.

**REFERENCES**


[14] D. Renga and M. Meo, “Dimensioning renewable energy systems to

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**Table 8:** Computation time (seconds) to find the cost-optimal system dimension using CECoDA and that using exhaustive search ([15], [16]), with $C_p = 0$, along with percentage saving in computation time

<table>
<thead>
<tr>
<th>City</th>
<th>CECoDA [15], [16]</th>
<th>CECoDA [15], [16]</th>
<th>CECoDA [15], [16]</th>
<th>CECoDA [15], [16]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p = 1%</td>
<td>191.69</td>
<td>2776.81</td>
<td>2657.14</td>
</tr>
<tr>
<td></td>
<td>saving = 93.11 %</td>
<td>281.64</td>
<td>2567.14</td>
<td>281.64</td>
</tr>
<tr>
<td></td>
<td>p = 0.1%</td>
<td>352.03</td>
<td>2782.32</td>
<td>2673.86</td>
</tr>
<tr>
<td></td>
<td>saving = 87.34 %</td>
<td>523.19</td>
<td>451.29</td>
<td>523.19</td>
</tr>
<tr>
<td></td>
<td>p = 0.01%</td>
<td>582.70</td>
<td>2783.41</td>
<td>2681.43</td>
</tr>
<tr>
<td></td>
<td>saving = 79.05 %</td>
<td>562.16</td>
<td>730.38</td>
<td>562.16</td>
</tr>
</tbody>
</table>

**Table 9:** Computation time (seconds) to find the cost-optimal system dimension using CECoDA and that using exhaustive search ([15], [16]), with $C_p = 10$, along with percentage saving in computation time

<table>
<thead>
<tr>
<th>City</th>
<th>CECoDA [15], [16]</th>
<th>CECoDA [15], [16]</th>
<th>CECoDA [15], [16]</th>
<th>CECoDA [15], [16]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p = 1%</td>
<td>175.87</td>
<td>2754.23</td>
<td>175.87</td>
</tr>
<tr>
<td></td>
<td>saving = 93.61 %</td>
<td>2754.23</td>
<td>175.87</td>
<td>2754.23</td>
</tr>
<tr>
<td></td>
<td>p = 0.1%</td>
<td>347.87</td>
<td>2767.69</td>
<td>175.87</td>
</tr>
<tr>
<td></td>
<td>saving = 87.42 %</td>
<td>2754.23</td>
<td>347.87</td>
<td>2754.23</td>
</tr>
<tr>
<td></td>
<td>p = 0.01%</td>
<td>541.28</td>
<td>2769.11</td>
<td>175.87</td>
</tr>
<tr>
<td></td>
<td>saving = 80.42 %</td>
<td>2754.23</td>
<td>541.28</td>
<td>2754.23</td>
</tr>
</tbody>
</table>

**Table 10:** Estimation of cost-optimal system dimension using CECoDA for two sets of cities of India for different energy outage constraint with $C_p = 0$

<table>
<thead>
<tr>
<th>City</th>
<th>Nashik</th>
<th>Pune</th>
<th>Aurangabad</th>
<th>Leh</th>
<th>Kargil</th>
<th>Srinagar</th>
</tr>
</thead>
<tbody>
<tr>
<td>p = 1%</td>
<td>(9.16)</td>
<td>(9.18)</td>
<td>(9.14)</td>
<td>(10.19)</td>
<td>(11.20)</td>
<td>(14.20)</td>
</tr>
<tr>
<td>p = 0.1%</td>
<td>(10.26)</td>
<td>(11.24)</td>
<td>(10.24)</td>
<td>(13.25)</td>
<td>(16.26)</td>
<td>(16.32)</td>
</tr>
<tr>
<td>p = 0.01%</td>
<td>(10.37)</td>
<td>(12.32)</td>
<td>(11.32)</td>
<td>(14.35)</td>
<td>(19.38)</td>
<td>(19.42)</td>
</tr>
</tbody>
</table>


[31] Huawei Technologies Corporation Ltd. eLTE3.1 DBS3900 LTE FDD Product Description, 2013, Issue 01.


APPENDIX A
PROOF OF THEOREM 1

Proof: To investigate the nature of multivariate function, it is important to analyze the Hessian matrix of the given function. If the Hessian matrix is positive semidefinite matrix in the domain of definition $(n_{PV} \geq n_{PV,B}, N_B \geq N_{Bb})$, then the function is convex function. If this condition is not satisfied then, the nature of eigenvalues of Hessian matrix is investigated.

The total expenditure is given as, $C_{Ex}(n_{PV}, N_B) = C_{CapEx}(n_{PV}, N_B) + C_{OpEx}(n_{PV}, N_B) + C_{ImpEx}(n_{PV}, N_B)$

where $C_{CapEx}(n_{PV}, N_B)$, $C_{OpEx}(n_{PV}, N_B)$, and $C_{ImpEx}(n_{PV}, N_B)$ are provided in (A.1), (A.2), and (A.3) respectively.

The Hessian matrix of total expenditure is given as, $H_{Ex} = H_{CapEx} + H_{OpEx} + H_{ImpEx}$

where $H_{CapEx}$, $H_{OpEx}$, and $H_{ImpEx}$ are the Hessian matrix of $C_{CapEx}(n_{PV}, N_B)$, $C_{OpEx}(n_{PV}, N_B)$, and $C_{ImpEx}(n_{PV}, N_B)$ respectively.

The Hessian matrix of $C_{CapEx}(n_{PV}, N_B)$ is given by, $H_{CapEx} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Therefore, the $C_{CapEx}(n_{PV}, N_B)$ is a convex function of $n_{PV}$ and $N_B$.

The Hessian matrix of $C_{OpEx}(n_{PV}, N_B)$ is given as, $H_{OpEx} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$.

where $h_{11}$, $h_{12}$, and $h_{21}$, and $h_{22}$ are given in (A.1), (A.2), and (A.3) respectively.

The variation of determinant of $H_{OpEx}$ for three different cities are shown in Fig. A.1. It can be noted that, $H_{OpEx}$ is not always positive over the domain of definition. Therefore, $C_{OpEx}(n_{PV}, N_B)$ is not a convex function of $n_{PV}$ and $N_B$.

On the other hand, the Hessian matrix of $C_{ImpEx}(n_{PV}, N_B)$ is given as, $H_{ImpEx} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Therefore, $C_{ImpEx}(n_{PV}, N_B)$ is a convex function of $n_{PV}$ and $N_B$.

Thus, the Hessian matrix of total expenditure is not positive semidefinite matrix in the domain of definition. Hence the cost function is not a convex function of $n_{PV}$ and $N_B$. To characterize it further, it is important to investigate the nature of eigenvalues of the Hessian matrix. If the Hessian matrix has at most one negative eigenvalue, then it is said to be a quasiconvex function. The plots of maximum and minimum of two eigenvalues are shown in Fig. A.2 and Fig. A.3 respectively. One can observe that, one eigenvalue is always positive in the domain of definition, whereas the other is either negative or positive. This proves quasiconvex nature of the total cost as a function of $n_{PV}$ and $N_B$.

APPENDIX B
PROOF OF THEOREM 2

Proof: The Hessian matrix of energy outage probability is given as, $H_{out} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$.

where $h_{11} = b_{10} \cdot e^{-n_{PV}} + b_{11} \cdot N_B \cdot e^{-n_{PV}} + 4 \cdot b_{20} \cdot e^{-2 \cdot n_{PV}}$, $h_{12} = -b_{11} \cdot e^{-n_{PV}}$, $h_{21} = -b_{11} \cdot e^{-n_{PV}}$, and $h_{22} = 2 \cdot b_{02}$.

The variation of determinant of $H_{out}$ for three different cities are shown in Fig. A.4. One can deduce that, the Hessian matrix is positive definite and hence the energy outage probability is a convex function of $n_{PV}$ and $N_B$. 

APPENDIX C
PROOF OF LEMMA 1

Proof: The total expenditure for a given panel size $n_{PV} = n_{PV_0}$ is expressed as, $C_{Ex}(n_{PV}, N_B) \big|_{n_{PV} = n_{PV_0}} = C_{PV} \cdot n_{PV_0} + C_B \cdot \frac{Y}{L(n_{PV_0}, N_B)} \cdot N_B + C_R \cdot n_{PV_0} \cdot A_{PV} \cdot Y + C_{all}$.

To investigate the nature of single variable function, it is important to investigate the nature of second derivative. The second derivative of $C_{Ex}(n_{PV_0}, N_B)$ with respect to $N_B$ is given in (C.1). The variation of $\frac{d^2 C_{Ex}(n_{PV_0}, N_B)}{d N_B^2}$ versus number of battery is shown in Fig. A.5 for different panel size in the domain of definition for different cities. The value of $\frac{d^2 C_{Ex}(n_{PV_0}, N_B)}{d N_B^2}$ is negative for some panel size. This indicates that, the $C_{Ex}(n_{PV_0}, N_B)$ is not a convex function of $N_B$.

Now, the first derivative of $C_{Ex}(n_{PV_0}, N_B)$ with respect to $N_B$ is given as, $\frac{d C_{Ex}(n_{PV_0}, N_B)}{d N_B} = L(n_{PV_0}, N_B) - N_B \cdot L_{N_0}(n_{PV_0}, N_B)$.

The variation of first derivative is shown in Fig. A.6 for different panel size and different cities. One can observe that, the first derivative changes sign only once for some panel size, whereas it does not change sign for some other panel sizes. This proves the quasiconvex nature of $C_{Ex}(n_{PV_0}, N_B)$ as a function of $N_B$ for a given panel size $n_{PV_0}$.

APPENDIX D
PROOF OF LEMMA 2

Proof: The second derivative of $P_{out}(n_{PV_0}, N_B)$ is given as, $\frac{d^2 P_{out}(n_{PV_0}, N_B)}{d N_B^2} = 2 \cdot b_{02}$.

One can observe from Table 5 that, $b_{02} > 0$ for all three cities, hence $P_{out}(n_{PV_0}, N_B)$ is a convex function of $N_B$. 

\[ h_{11} = \frac{-L^2(n_{PV}, N_B) \cdot N_B \cdot L_{n_{PV}n_{PV}}(n_{PV}, N_B) + 2 \cdot N_B \cdot L(n_{PV}, N_B) \cdot L_{n_{PV}}(n_{PV}, N_B)}{L^4(n_{PV}, N_b)} \]

with 
\[ L_{n_{PV}}(n_{PV}, N_B) = \frac{\partial L(n_{PV}, N_B)}{\partial n_{PV}} = a_{10} + a_{11} \cdot n_{PV} + 2 \cdot a_{20} \cdot n_{PV}^2 + 2 \cdot a_{21} \cdot N_B + a_{12} \cdot N_B^2 + 3 \cdot a_{30} \cdot n_{PV}^3, \]
\[ L_{n_{PV}n_{PV}}(n_{PV}, N_B) = \frac{\partial^2}{\partial n_{PV}^2} L(n_{PV}, N_B) = 2 \cdot a_{20} + 2 \cdot a_{21} \cdot N_B + 6 \cdot a_{30} n_{PV}. \]

(A.1)

\[ h_{12} = h_{21} = \frac{-L^2(n_{PV}, N_B) \cdot \left[ L_{n_{PV}}(n_{PV}, N_B) - N_B \cdot L_{n_{PV}n_{PV}}(n_{PV}, N_B) \right]}{L^4(n_{PV}, N_b)} + 2 \cdot N_B \cdot L_{n_{PV}}(n_{PV}, N_B) \cdot L(n_{PV}, N_B) \cdot L_{n_{PV}n_{PV}}(n_{PV}, N_B), \]

with 
\[ L_{n_{PV}n_{PV}}(n_{PV}, N_B) = \frac{\partial^2}{\partial n_{PV} \partial N_B} L(n_{PV}, N_B) = a_{11} + 2 \cdot a_{21} \cdot n_{PV} + 2 \cdot a_{12} \cdot N_B. \]

(A.2)

\[ h_{22} = \frac{-L^2(n_{PV}, N_B) \cdot \left[ L_{N_B}(n_{PV}, N_B) - N_B \cdot L_{N_Bn_{PV}}(n_{PV}, N_B) - L_{N_Bn_{PV}}(n_{PV}, N_B) \right]}{L^4(n_{PV}, N_b)} - \frac{\partial}{\partial N_B} L(n_{PV}, N_B) = a_{01} + a_{11} \cdot n_{PV} + 2 \cdot a_{02} \cdot N_B + a_{21} \cdot n_{PV}^2 + 2 \cdot a_{12} \cdot N_B^2 + 3 \cdot a_{03} \cdot N_B^2, \]
\[ L_{N_B}(n_{PV}, N_B) = \frac{\partial^2}{\partial N_B^2} L(n_{PV}, N_B) = 2 \cdot a_{02} + 2 \cdot a_{12} \cdot n_{PV} + 6 \cdot a_{03} \cdot N_B. \]

(A.3)

\[ \frac{d^2 C_{Ex}(n_{PV}, N_b)}{d N_B^2} = \frac{-L^2(n_{PV}, N_B) \cdot \left[ L_{N_B}(n_{PV}, N_B) - N_B \cdot L_{N_Bn_{PV}}(n_{PV}, N_B) - L_{N_Bn_{PV}}(n_{PV}, N_B) \right]}{L^4(n_{PV}, N_B)} - \frac{\partial}{\partial N_B} L(n_{PV}, N_B) \cdot L_{N_B}(n_{PV}, N_B) \cdot L_{N_Bn_{PV}}(n_{PV}, N_B) \cdot L_{N_Bn_{PV}}(n_{PV}, N_B). \]

(C.1)

where 
\[ L_{N_B}(n_{PV}, N_B) \text{ and } L_{N_Bn_{PV}}(n_{PV}, N_B) \text{ can be obtained from } (A.3) \text{ with } n_{PV} = n_{PV0}. \]

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**Figure A.1:** Variation of \( |H_{OpEx}| \) versus panel size and number of battery units, for different cities.

---

**Figure A.2:** Variation of maximum of eigenvalues versus panel size and number of battery units, for different cities.
Figure A.3: Variation of minimum of eigenvalues versus panel size and number of battery units, for different cities.

Figure A.4: Variation of $|H_{out}|$ versus panel size and number of battery units, for different cities.

Figure A.5: Variation of second derivative versus number of battery units for different panel size, for different cities.

Figure A.6: Variation of first derivative versus number of battery units for different panel size, for different cities.